Limited information and the relation between the variance of inflation and the variance of output in a new keynesian perspective.

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Abstract

When the central bank minimizes a quadratic loss function depending upon the inflation gap and the output gap, a negative association between the variance of inflation and the variance of output emerges. The variance of output will be higher the greater is the preference of the central bank for stabilizing inflation. Instead, when the central bank sets the interest rate according with the minimization problem, but on ahead of the beginning of the correspondent period, the tradeoff between the variance of inflation and the variance of output disappears completely.

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INTRODUCTION

John Taylor has made enormous contributions to macroeconomics. In 1979 (Taylor (1979)) he asserted that instead of a long run tradeoff between inflation and unemployment, there was a long run tradeoff between the variance of inflation and the variance of output (Taylor (1979 p. 1280)). Policies oriented to reduce the variance of inflation were likely to generate higher variance in output.

Taylor (1994) retakes the negative relation already discussed, but says that empirically there is not a clear negative relation between the variance of inflation and output. He attributes this fact to the use of inefficient monetary policies. When the central bank uses ad-hoc interest rate policy rules, it is far from an efficient negative frontier between the variance of inflation and the variance of output. Then, there may be situations where both variances do not have any definite relation or in fact they may be correlated positively.

According to Taylor (1994) and Svenson (2010) and efficient monetary policy is one that has inflation targeting as a necessary condition (though not sufficient), a practice that started in New Zealand in 1990 and that has been adopted by numerous countries since then.

This paper analyzes a case where an efficient monetary policy generates a situation where the variance of inflation and the variance of output are not related. This may occur when, besides the usual constraints, the central bank faces an institutional constraint of setting interest rates at discrete points in time. In a theoretical framework this happens when the central bank sets interest rates at the beginning of certain period, or at the end of the last period. In this case it is impossible to foresee random supply and demand shocks that will take place later in the period. That situation may break the negative theoretical relation defined originally by Taylor (1979) (1994).
The paper has two main sections: the first one show the “classical” new Keynesian solution proposed by Taylor, where there is a definite negative frontier between the variance of inflation and the variance of output. Second section shows the alternative solution, where the central bank sets interest rates at the end of the last period. In this case there is total independence of the variance of inflation and the variance of output, since these variables do not depend of monetary policy parameters. May this explanation be the cause why different studies do not find the negative relation proposed by the new Keynesian school? That is something that has to be analyzed in the coming future.¹


In the simplest new Keynesian model prices are staggered (Taylor (1980), Calvo (1983)), which generate the new Phillips curve. There is an old IS curve linking output and the real rate of interest negatively and the central bank minimizes a loss function. This one depends upon the quadratic deviations of inflation from its target (the inflation gap or the inflation cycle) and output from its normal or potential level (the output gap).

The simplest new Phillips curve is the one derived by Mankiw and Reis (2001) based in the work by Calvo (1983), which may be described as:

\[ \pi_t = E_t \pi_{t+1} + \delta(y_t - y^*) + e_t \quad (1) \]

Where \( \pi_t \) is the actual rate of inflation; \( E_t \pi_{t+1} \) is the conditional expected value of inflation in the next period. The variable \( y_t \) is present output and \( y^* \) is potential output. Therefore \( y_t - y^* \) is the output gap. The term \( e_t \) is an independent uncorrelated normally distributed supply random shock with zero mean and constant variance \( \sigma^2 \).

The IS curve in the economy is defined as.

\[ y_t = H - br_t + v_t \]  \hspace{1cm} (2)

This function establishes a consistent negative relation between income \( y \) and the real rate of interest \( r \). Parameter \( v \) is an independent uncorrelated random shock normally distributed with zero mean and variance equal to \( \sigma_v^2 \).

Parameter \( H \) could represent a term related to fiscal policy parameters, where increases in government expenditure increases \( H \) \((dH/dG>0)\) or increases in the income tax rate reduces \( H \) \((dH/dt<0)\). Other variables from the private sector, like autonomous expenditure, may be also in the parameter \( H \) of the traditional IS curve.

In the new Keynesian “classical” approach, the central bank has a well-defined objective, which in many cases consist of minimizing the following loss function (see for example Taylor (1979)).

\[ L_t = \varphi (\pi_t - \pi^*)^2 + (1 - \varphi) (y_t - y^*)^2 \]  \hspace{1cm} (3)

Where \( \pi^* \) is the targeted inflation. \((\pi_t - \pi^*)\) is the inflation gap or the inflation cycle.

The loss function is quadratic as in the Barro-Gordon approach (Barro and Gordon (1983)).\(^2\) The best possible outcome for the central bank is to have actual inflation \( \pi_t \) equal to the inflation target \( \pi^* \) and present output \( y_t \) equal to natural output \( y^* \). Every other possibility produces a positive loss function.

The parameter \( \varphi \) is a representation of how important is for the central bank the stability of inflation around its target in comparison to the stability of output \((0 < \varphi < 1)\). When \( \varphi \) is 1 the central bank is just committed to set inflation in its target. On the contrary, when \( \varphi=0 \) \((1-\varphi =1)\) the central bank just cares about the stability of output. In practice the central bank cares about the two objectives.\(^3\)

The primal problem to solve is to minimize equation (3) of the loss function subject to equation (1) of the Phillips curve. That will generate a relation between the output gap \((y_t-\)

\(^2\) An important difference of this approach with the Barro-Gordon version is that in the last one the target for income is greater than the natural output \( y^* \).

\(^3\) Svenson (2010) argues that while some central banks have the unique commitment of low inflation, in practice all of them care also about output.
\(y^*\) and the inflation gap \((\pi_t - \pi^*)\). To get this function the central bank has to set a monetary rule for the rate of interest, which is the very well-known Taylor rule (see Taylor (1993)).

Minimization of (3) subject to (1) gives, as a result:

\[
y_t - y^* = - \frac{\phi \delta (E_t \pi_{t+1} - \pi^*)}{(\phi \delta^2 + (1 - \phi))} - \frac{\phi \delta e_t}{(\phi \delta^2 + (1 - \phi))} \quad (4)
\]

This equation, which we call the aggregate demand, shows a negative association between the output gap \((y_t - y^*)\) and a proxy of the inflation gap \((E_t \pi_{t+1} - \pi^*)\).

For equation (4) being in fact the aggregate demand, the central bank has to set a rule for interest rates. Substituting (4) in the IS curve (2) we get:

\[
r_t = \frac{H - y^*}{b} + \frac{\phi \delta (E_t \pi_{t+1} - \pi^*)}{(\phi \delta^2 + (1 - \phi))b} + \frac{\phi \delta e_t}{(\phi \delta^2 + (1 - \phi))b} + \frac{\nu_t}{b} \quad (5)
\]

Equation (5) is the famous Taylor rule in real interest rates (the MP equation of the Romer-Taylor model. See Romer (2000), Taylor (2000)). When inflation and its expectations are stable \(E_t \pi_{t+1} = \pi^*\), and in absence of random shocks coming from supply and/or demand:

\[
r_t = \frac{H - y^*}{b} = r^* \quad (6)
\]

Where \(r^*\) is the natural rate of interest, a term coined more than one hundred years ago by the Swedish economist Knut Wicksell (Wicksell (1898)).

In the traditional IS-LM analysis (Hicks (1937)), the natural rate of interest depends upon the parameter \(H\), which instead is driven in part by fiscal policy. Higher government expenditure, or lower income tax rate, are factors that increase the natural rate of interest and then, through the Taylor rule (5), the actual real rate of interest.\(^4\)

The dual problem consists of taking the Taylor rule (5) and substitute it in the IS curve (2). The result is the aggregate demand (4). Clearly, the Phillips curve (1) and the aggregate

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\(^4\) A very different result surges when the IS comes from consumer optimization (see McCallum and Nelson (1999) or King (2000)) in this case the natural rate of interest is the subjective rate of discount of the utility of consumers.
demand (4) are the relevant equations to solve for inflation ($\pi_t$) and output ($y_t$), while the IS curve solves for the real interest rate $r_t$.\(^5\)

The way to solve for inflation and output is to substitute the output gap ($y_t - y^*$) of the aggregate demand equation (4) in the Phillips curve (1), which gives, as a result:

\[
\pi_t = \frac{(1-\phi)}{\phi \delta^2 + (1-\phi)} E_t \pi_{t+1} + \frac{\phi \delta^2}{\phi \delta^2 + (1-\phi)} \pi^* + \frac{(1-\phi)}{\phi \delta^2 + (1-\phi)} e_t
\]

Inflation depends upon its future expectations, the inflation target and the supply shock $e_t$. In absence of random shocks inflation would be a weighted average of its future expectation and its target.

To solve analytically the difference equation (7) we use the forward operator (see Hamilton (1994)).

\[
E_t \pi_{t+1} = L^{-x} \pi_t \quad (8)
\]

Also, to simplify notation, we call

\[
j = \frac{\phi \delta^2}{(\phi \delta^2 + (1-\phi))} \quad (9)
\]

\[
1 - j = \frac{(1-\phi)}{(\phi \delta^2 + (1-\phi))} \quad (10)
\]

Solution for inflation is then

\[
\pi_t = \pi^* + (1 - j) \sum_{i=0}^{\infty} (1 - j)^i E_t e_{t+i} \quad (11)
\]

But

\[
\sum_{i=0}^{\infty} (1 - j)^i E_t e_{t+i} = e_t \quad (12)
\]

Since $E_t e_{t+i} = 0$ for $i > 0$

Then the definite reduced form for inflation is

\[^5\text{It is interesting to note that the form of the IS curve is irrelevant in the solution of inflation and output. We could be using a new IS curve, like the one used in McCallum and Nelson (1999), King (2000) and Blanchard (2008), and the results for inflation and output and their variances would be exactly the same.}\]
\[ \pi_t = \pi^* + (1 - j)e_t \quad (13) \]

At the same time, to solve for \( y \) it is possible to set aggregate demand (4) as:

\[ y_t - y^* = -\frac{j}{\delta}(E_t\pi_{t+1} - \pi^*) - \frac{j}{\delta}e_t \quad (14) \]

But using (13) it is possible to show that:

\[ E_t\pi_{t+1} = \pi^* \quad (15) \]

Therefore, the reduced form for output is:

\[ y_t = y^* - \frac{j}{\delta}e_t \quad (16) \]

Positive supply shocks \((e_t<0)\) generate a situation where output rises above the natural level and inflation falls. However, given the assumptions of the model the impact of these shocks dilutes in just one period.

There is a tradeoff between the variance of output and the variance of inflation. Output will be more variant the more worried is the central bank maintaining inflation around the target \( \pi^* \) and vice versa.

The non-conditional expectation for inflation in equation (13) is \( \pi^* \), since the non-conditional expectation for the disturbance term \( e_t \) is zero. Therefore, the variance of inflation is:

\[ Var(\pi_t) = E(\pi_t - E(\pi_t))^2 = E(\pi_t - \pi^*)^2 = (1 - j)^2\sigma^2 \quad (17) \]

At the same time, the non-conditional expectation for output in (16) is \( y^* \) because of the same reason that in the previous equation. Then

\[ Var(y_t) = E(y_t - E(y_t))^2 = E(y_t - y^*)^2 = \frac{j^2}{\delta^2}\sigma^2 \quad (18) \]

Parameter \( j \) is directly related to \( \varphi \), which instead represents how much the central bank cares about the variance of inflation compared with the variance of output. When \( \varphi=1 \), then \( j=1 \) and the central bank cares just about maintaining inflation in its target \( \pi^* \). When \( \varphi=0 \), \( j=0 \) and the central bank cares just about to maintain output at its natural level \( y^* \).
There is a monotonic relation between \( j \) and \( \varphi \), which can be seen taking the derivative of \( j \) with respect to \( \varphi \) in (9):

\[
\frac{dj}{d\varphi} = \frac{\delta^2}{(\varphi \delta^2 + (1-\varphi))^2} > 0 \quad (19)
\]

\[
\frac{d\text{Var}(\pi_t)}{dj} = -2(1-j)\sigma^2 < 0 \quad (20)
\]

\[
\frac{d\text{Var}(y_t)}{dj} = 2 \frac{j}{\delta^2} \sigma^2 > 0 \quad (21)
\]

The more the central bank cares to maintain inflation in its target \( \pi^* \) (the higher are \( \varphi \) and \( j \)), the lower is the variance of inflation and the higher is the variance of output. There is a tradeoff between the variance of inflation and the variance of output.


The highest possible variance of output and the lowest possible variance for inflation occur when \( \varphi=1=j \), in which case:

\[
\text{Var}(\pi_t) = 0 \quad (22)
\]

\[
\text{Var}(y_t) = \frac{\sigma^2}{\delta^2} \quad (23)
\]

The highest possible variance of inflation and the lowest possible variance for output occur when \( \varphi=0=j \), when

\[
\text{Var}(\pi_t) = \sigma^2 \quad (24)
\]

\[
\text{Var}(y_t) = 0 \quad (25)
\]

The variance’s function is a relation between the variance of inflation and the variance of output, which is found solving for \( j \) in (18) and substituting in (17), which gives, as a result:

\[
\text{Var}(\pi_t) = \sigma^2 - 2(\text{Var}(y_t))^{1/2} \delta \sigma + \text{Var}(y_t)\delta^2 \quad (26)
\]
It is possible to prove that this function has a negative slope, since the first derivative of the variance of inflation with respect to the variance of output is:

$$\frac{d(Var(\pi_t))}{d(Var(y_t))} = -\left(Var(y_t)\right)^{-\frac{1}{2}}\delta \sigma + \delta^2 \quad (27)$$

For this derivate being negative, it is necessary to have:

$$-\left(Var(y_t)\right)^{-\frac{1}{2}}\delta \sigma + \delta^2 < 0 \quad (28)$$

But that means:

$$\frac{\sigma^2}{\delta^2} > Var(y_t) \quad (29)$$

This is true because the maximum variance of y is exactly $\sigma^2/\delta^2$.

At the same time, the second derivative is positive, showing that there is a convex relation in the plane where the variance of inflation is in the vertical axis and the variance of output is in the horizontal axis.

This can be seen taking the second derivative in (27).

$$\frac{d^2Var(\pi_t)}{d(Var(y_t))} = \frac{1}{2} \left(Var(y_t)\right)^{-3/2}\delta \sigma > 0 \quad (30)$$

Which implies that in the plane where the variance of inflation is in the vertical axis, and the variance of output is in the horizontal axis, the relation between these two variables is inverse and convex to the origin.

On the other hand, the expected value of the loss function (3) is a linear relation between the variance of inflation and the variance of output, which may be seen applying the non-conditional expectation’s operator to equation (3) and considering that the non-conditional expectation for inflation and output are $\pi^*$ and $y^*$, respectively.

$$E(L_t) = \phi E(\pi_t - \pi^*)^2 + (1 - \phi)E(y_t - y^*)^2 = \phi E(\pi_t - E(\pi_t))^2 + (1 - \phi)E(y_t - E(y_t))^2 = \phi Var(\pi_t) + (1 - \phi)Var(y_t) \quad (31)$$
The expected result for the variance of inflation and the variance of output occurs in the tangency between the variance’s function (26) and the expected loss function (31). While there is a unique variance’s function, there is a dense map of expected loss functions, all of them linear.

Figure 1: The tradeoff between the variance of inflation and the variance of output in the “classical” new Keynesian model.

As we have seen, to minimize the loss function, the central bank has to set an interest rate rule, the famous Taylor rule (equation (5)) (Taylor (1993)). When the central bank foresees all possible supply and demand shocks that will occur during the period, the Taylor rule includes those shocks. We repeat equation (5) for convenience.

\[ r_t = \frac{1}{b} (H - y^* + \frac{\phi \delta}{(\phi \delta^2 + (1-\phi))} (E_{t-1} \pi_{t+1} - \pi^*) + \frac{\phi \delta}{(\phi \delta^2 + (1-\phi))} e_t + \nu_t) \]  

(32)

The optimizing Taylor rule (32) has a form very similar to the one proposed by Taylor (1993).

In real life information is limited. Central banks set interest rates at discrete points of time, usually in a meeting of the board of governors. Assuming that periods last a certain symmetric interval, for instance three months, it is clear that if the bank sets the interest rate at the beginning of the period, or at the end of last period, it may not be able to forecast at least some of the supply and demand shocks that will occur during the future months of the period. It is equally true that if shocks are not very severe, the bank will wait until the next programmed meeting of the board of governors to consider a modification of the interest rate.

In this section we will assume the other extreme case than in the first section. Central banks set interest rates for period \( t \) at period \( t-1 \). They do not know at that time which supply and demand shocks will prevail in \( t \). Then, in fact, the best forecast to do at that moment is zero for all shocks.

Usually, central banks set nominal interest rates instead of real interest rates. Given the information that the central bank has in period \( t-1 \), the best proxy of the optimal rule (32) would be to set the nominal interest rate in the following way:

\[ R_t = E_{t-1} \pi_{t+1} + \frac{1}{b} (H - y^* + \frac{\phi \delta}{(\phi \delta^2 + (1-\phi))} (E_{t-1} \pi_{t+1} - \pi^*)) \]  

(33)

Where \( R \) is the nominal interest rate that will prevail en \( t \) but setting at the end of \( t-1 \).\(^6\)

The definition of the real interest rate is, however:

\(^6\) In the strict sense of the model \( R_t \) is set in \( t-1 \). It is not possible algebraically to distinguish between the beginning and the end of one period in discrete time.
\[ r_t = R_t - E_t \pi_{t+1} \quad (34) \]

Then, substituting equations (33) and (34) in the IS (equation (2)) we get

\[ y_t = y^* - \frac{\varphi \delta}{(\varphi \delta^2 + (1 - \varphi))} (E_{t-1} \pi_{t+1} - \pi^*) - b(E_{t-1} \pi_{t+1} - E_t \pi_{t+1}) + v_t \quad (35) \]

Which is an aggregate demand slightly different to the function (equation (4)) set in the minimization problem of the first section.

Substituting this aggregate demand (35) in the new Phillips curve (1) we get:

\[ \pi_t = E_t \pi_{t+1} - jE_{t-1} \pi_{t+1} + j \pi^* - b \delta(E_{t-1} \pi_{t+1} - E_t \pi_{t+1}) + \delta v_t + e_t \quad (36) \]

Where \( j \) is defined in exactly the same way than in equation (9).

Taking the expectation in \( t-1 \) of equation (36) and using the law of iterative expectations, where

\[ E_{t-1} E_t \pi_{t+1} = E_{t-1} \pi_{t+1} \quad E_{t-1} E_{t-1} \pi_{t+1} \quad (37) \]

We get

\[ E_{t-1} \pi_t = (1 - j) E_{t-1} \pi_{t+1} + j \pi^* \quad (38) \]

We can solve this difference equation either by recursive substitution or using the forward operator

\[ E_{t-1} \pi_{t+1} = L^{-1} E_{t-1} \pi_t \quad (39) \]

Which means

\[ E_{t-1} \pi_t = \pi^* \quad (40) \]

But it is also true that in period \( t+1 \)

\[ E_t \pi_{t+1} = (1 - j) E_t \pi_{t+2} + j \pi^* \quad (41) \]

If we take expectations of equation (41) conditional in the information in \( t-1 \), and considering the law of iterative expectations:
\[ E_{t-1}\pi_{t+1} = (1 - j)E_{t-1}\pi_{t+1} + j\pi^* \] (42)

Therefore, applying a forward operator similar to the one shown in (39)

\[ E_t\pi_{t+1} = \pi^* \] (43)

\[ E_{t-1}\pi_{t+1} = \pi^* \] (44)

Substituting (43) and (44) in (36) we get the reduced form for inflation

\[ \pi_t = \pi^* + \delta v_t + e_t \] (45)

Also, substituting (43) and (44) in the aggregate demand (35), we get the reduced form for output.

\[ y_t = y^* + v_t \] (46)

Since we assume that v and e are uncorrelated and independent normal shocks with zero mean:

\[ Var(\pi_t) = E(\pi_t - E\pi_t)^2 = E(\pi_t - \pi^*)^2 = \delta^2\sigma_v^2 + \sigma^2 \] (47)

\[ Var(y_t) = E(y_t - Ey_t)^2 = E(y_t - y^*)^2 = \sigma_v^2 \] (48)

In this case, the conditional and unconditional expectation of inflation and output are, respectively, the objective inflation \( \pi^* \) and the natural output \( y^* \). This result is almost identical to the one obtained in section I. However, the variances of inflation and output are different to the case where there was perfect information. When the Taylor rule for period \( t \) is set before that time, the variances of inflation and output are not related at all with the preferences of the central bank. Therefore, there is not any tradeoff between these variances when these preferences change.

It is possible to show that the limited information generates an inefficient solution with respect to the case described in section I.\(^7\) Suppose the extreme case where \( \sigma_v^2 \)- the variance of the IS random shocks- is equal to zero. In that case the variance of inflation would be \( \sigma^2 \) and the variance of output would be zero. That point is over the frontier of variances of

\(^7\) The perfect information case
inflation and output in the efficient case seen in figure 1 of section I. If $\sigma_v^2$ is positive, the usual case- then in the graph where the variance of inflation is in the vertical axis and the variance of output is in the horizontal axis, the point where these variances occur is at the north-east of the point where the variance of inflation is $\sigma^2$ and the variance of output is zero. That is to say the point is ahead of the efficient frontier and for that reason is inefficient.

The central bank cannot incorporate the supply and demand shocks occurring in period t in the interest rate rule of that time, since the rule is set just before. The common idea that targeting inflation produces a tradeoff between the variance of inflation and the variance of output is challenged by this example.

**CONCLUSIONS**

New Keynesian economics has popularized a new tradeoff, one where the variance of inflation is inversely related to the variance of output. Central banks more committed to hold the target of inflation than to maintain output near its natural level, will generate higher variance of output and lower variance of inflation, and the other way around.

Empirical works show mixed results: Bean (1998) estimates a negative frontier between the variance of inflation and the variance of output for the UK. Cecchetti and Ehrman (1999) find the same negative association for a sample of countries targeting inflation. Lee (1999) finds a weak negative association between the variances in question. On the other hand, Taylor (1994) asserts that is difficult to find empirical observations where the theoretical negative association already analyzed exists. He attributes that to the fact that monetary policies have been inefficient. Ball and Sheridan (2005) do not find more variation in output in the presence of inflation targeting and a reduction of the variance of inflation. Similar results are found by Batini and Laxton (2007) and Goncalves and Salles (2008).

In this paper we show that lack of information on the part of the central bank breaks the theoretical negative association between the variance of inflation and the variance of output. The central bank sets interest rates at discrete points on time. If the setting of this rate occurs before the emergence of a supply or demand shock, this one cannot be
considered in the setting of the interest rate of the correspondent period. Then, the way in which these shocks affect the variance of inflation and output may be very different to the case in which the central bank foresees the shocks.

The present work shows two extreme cases: one where in within the period the central bank foresees all possible supply and/or demand shocks. The other one where the central bank cannot see any of the future shocks that will take place in the correspondent period. In the first case, there is a definite negative relation between the variance of inflation and the variance of output; in the second one the relation disappears completely.

Reality is in between these two cases. In the day of the setting of the interest rate for the period in question, the central bank may foresee some possible shocks; for instance, if the weather is deteriorating, it will be possible to foresee hurricanes or other situations that constitute negative supply shocks. Instead, the weather may be nice in the day of the setting of the interest rate but soon after that may be deterioration and negative supply shocks take place.

It may be argued that the central bank should have a contingent policy, where in case of shocks the rate of interest changes. However, in reality possible changes in the interest rate are many times scheduled and unforeseen shocks occur in between the meetings of the board of governors of the bank. Also, the central bank does not want to be changing the nominal rate of interest quite frequently in order to send a signal of order to financial markets. If the severity of the shocks is not very high, the monetary authority prefers to maintain the nominal rate of interest unchanged.

The lack of information about shocks, and not necessarily the presence of inefficient monetary policies that do not consider inflation targeting, as Taylor (1994) asserts, may be an important factor of the breaking of the theoretical negative relation between the variance of inflation and the variance of output.
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