Learn, sweat or steal: a theory of development and the activity of children

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Diciembre 2013
Documento de trabajo No. 06, 2013
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November 2013

Abstract

I explore the effects credit market imperfections and institutional quality –security of property rights and quality of education– have on the joint distribution of schooling, child labor and child crime in developing countries. To that end, I develop and analyze an overlapping generations model of endogenous growth and inequality in the distribution of human capital. In this context, I argue that banning child labor permanently in countries with poor institutions unambiguously harms the working children living in the poorest households. The insight is that the poor institutional quality leads the poorest households to face a tradeoff for their child’s activities not between work and school, but between work and crime. Accordingly, the ban leads those children into crime. In turn, once these children are adults, their own children will be involved into crime, and so on, negatively affecting everyone in the long run. Furthermore, even a temporary ban on child labor can have a permanent negative effect on everyone’s welfare if the quality of education is sufficiently poor. Finally, I argue that investing on the institutional quality of the economy benefits all generations of children if this investment is financed by those households with the largest levels of income.

JEL Classification: O10, E24, J20, K42.

Keywords: child labor, child crime, economic development, enforcement of compulsory schooling, quality of education.

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1 Introduction

The goal of deterring child labor is enshrined in countless agreements and human rights documents signed in most of countries around the world. Nonetheless, the latest estimates of the ILO indicate that at least 215 million children are involved in child labor (14% of the global child population). Conventional views call for the elimination of child labor practices to ensure the welfare of the children, who are affected both cognitively and physically by the often repetitive and demanding nature of the work. However, the issues involved in deterring child labor are not as clear-cut as they may initially appear. When implementing policies to deter child labor, countries and organizations rarely take into account the fact that children often work to obtain much-needed resources for their families and when these well-intentioned policies render hazardous jobs out of bounds for children, the children themselves are driven into the underground economy where they are not protected at all, resulting in highly counterproductive outcomes. For children in the poorest conditions, school is not necessarily an option. Under certain circumstances, such as extreme poverty or lack of additional opportunities, households often face a tradeoff not between work and school, but between work and other activities to obtain resources such as child crime and prostitution. An example of this occurred in the garment industry in Bangladesh in 1993, where about 50,000 children were removed from the industry. Visits conducted by UNICEF found that children subsequently began engaging in activities such as prostitution, hustling, and stone crushing (UNICEF, 1997, p. 60).

In this paper I argue that in economies characterized by low institutional quality and human capital inequality, banning permanently child labor can harm the development of the children living in poor households and their entire dynasties. Furthermore, if the quality of education is sufficiently low, even a temporary ban on child labor can have permanent negative effects not only for the current generation of the poor children, but also for all individuals in the economy in the long run.

To this end, I develop and analyze an overlapping generations model of endogenous growth and inequality in the distribution of human capital. The model formalizes the fact
that in developing countries where the institutional quality is low, it is easier for children
to access illegal activities as a means to obtain resources needed for consumption. This
study sheds light on the short run and long run consequences, for both the children and the
economy in aggregate, of poor institutional quality and inequality in the economy.

The analysis addresses the short-run and long-run consequences of a permanent and a
temporary ban on child labor across different households. In addition, I examine how three
important aspects of institutional quality – the security of property rights, the quality of
education, and the access to credit markets – affect the tradeoff different households face
when choosing whether to have their children attending school, working, or involved in crime.
I argue that even without credit market imperfections, child labor can still be present in the
economy. Furthermore, I show that having access to credit markets may have no effect on
schooling decisions if the productivity of child labor is sufficiently poor. However, a sufficient
increase in school quality may eliminate child labor even in the presence of credit market
imperfections. Finally, the analysis also illustrates how the activity of the children is affected
by differences in inequality.

As in Gonzalez and Rosales (2013), I consider that child labor and child crime are two
competing uses of a child’s time and both are a source of current household income. In
addition, I also consider that crime is a non-productive activity that harms the human
capital of the child. However, the model presented here differs in two important ways,
leading to new results. First, I depart from the symmetric equilibrium analysis and consider
that individuals are different. The focus is on the incentives different households have when
allocating their children’s time, and their implications for policy. In particular, I assume the
economy is characterized by inequality on the distribution of human capital. I incorporate
this assumption into an endogenous growth model with human capital accumulation as the
engine of growth. The specification of the human capital accumulation structure used here
allows examining the role played by the quality of education in the economy on the activity
of the children. Second, I assume that households have access to credit markets in the sense
that, if necessary, households can borrow resources. This assumption allows identifying the
effects of credit market imperfections.
I show that a child’s activity is determined by the income of the household she lives in. Children living in the poorest households are involved in crime, while children living in households with the highest levels of income attend school full time. Within the intermediate levels, the children of those households with relatively low levels of income work full time, and the children of those households with relatively large levels of income combine their time between school and work. In the long run, the distribution of human capital is an endogenous outcome that is a function of the quality of education and the security of property rights. This sorting is consistent with the evidence found in different studies. For instance, studies conducted by the ILO show that school enrolment is determined by poverty, and the accessibility, quality and cost of education. These studies also show that poverty leads children to alternative mechanisms of subsistence, such as crime, mainly to help their families.¹

In order to examine the effects of a permanent ban and a temporary ban on child labor I consider two cases: i) a small open economy and ii) a closed economy.

In the first case, I show that a permanent ban on child labor can have permanent negative effects for the children living in the poorest households if the level of property rights in the economy is sufficiently low. The ban leads some of the poor households to involve their children in crime, when otherwise they would have preferred to send their children to work. Consequently, the ban on child labor directly affects the development of a child living in poor households, by reducing her accumulation of human capital. In turn, this leads the affected children to involve their own children in crime in the future. Moreover, I show that even a temporary ban on child labor can have permanent negative effects for the children living in the poorest households if the quality of education and the level of property rights in the economy are sufficiently low. Intuitively, economies that prematurely implement policies to ban child labor exacerbate the problem of child crime whenever property rights and quality of education are sufficiently poor, reducing the accumulation of human capital and physical capital in the long run.

In the second case I show that, in the short run, a ban on child labor harms the children

¹See, for instance, ILO (2002a), ILO (2002b), and Dowdney (2003).
living in the poor households and benefits the children living in the rich households. Furthermore, I show an interesting result that has not been previously identified: if the quality of education and the level of property rights in the economy are sufficiently low, even a temporary ban on child labor can have permanent negative effects for all individuals in the economy. This result is due to the fact that, in the closed economy case, the temporary ban not only affects the households in the poorest conditions by driving children into crime, but it also decreases the physical capital-human capital ratio of the economy in the long run, reducing in turn the labor income and consumption of all individuals.

Next, in the context of the model, I examine the relationship between economic institutions and the activity of children. First, I show that an exogenous increase in either the quality of education or the security of property rights increases the proportion of children attending school full time, and it decreases the proportion of children out from school. Following this result, I examine the effects of a policy designed to increase the institutional quality in the economy financed through taxes. The results suggest that it is possible that investing on institutional quality might result in lower proportions of children working full time and children involved in crime. However, for this to occur, certain conditions must hold. Specifically, either the institutional quality should be very sensitive to the investment, or the costs associated with the investment on institutional quality should be covered by those households with the largest levels of income.

Second, I consider the case of credit market imperfections where the resources households can borrow from the capital markets are restricted to a certain level. I show that credit market imperfections might decrease the proportion of children attending school full time, increasing in turn the proportion of children combining school and work. In particular, this occurs only if the productivity of child labor is sufficiently large.

Finally, the implications of differences in inequality are discussed. In order to measure inequality, the approach used here is based on the concept of second-order stochastic dominance. In this case, I argue that the long-run level of child crime is greater in economies with larger inequality. In turn, this implies that the growth rate of economies with larger inequality will be lower during the transition to the steady state equilibrium.
There is a growing literature examining the causes and consequences of child labor.\footnote{Basu (1999), Basu & Tzannatos (2003), and Edmonds (2008) are excellent surveys of this literature.} In the theoretical work it is well established that child labor is a persistent phenomenon associated primarily with poverty and credit market imperfections (Basu & Van, 1998; Baland & Robinson, 2000; Ranjan, 2001). It has been recognized that policies to control child labor may cause child labor to rise (Basu, 2005; Basu & Zarghamee, 2009; Baland & Duprez, 2009), or a household’s welfare to fall if the loss in child labor earnings is not compensated by an equivalent increase in adult labor income (Basu & Van, 1998; Dessy & Pallage, 2005). Different studies have also analyzed the effects of child labor restrictions through international labor standards (Basu et al., 2003), and the endogenous adoption of child labor laws from a political economy perspective (Doepke & Zilibotti, 2005; Dessy & Knowles, 2008; Doepke & Zilibotti, 2010). Few studies have focused on the hazardous forms of child labor, in which policy interventions in the hazardous sector might be undesirable (Dessy & Pallage, 2005), or welfare improving (Rogers & Swinnerton, 2008).\footnote{Dessy & Pallage (2005) argue that banning child labor in the hazardous sector may be undesirable since this sector maintains child labor wages in the good sector high enough to allow for human capital accumulation. In a model of exploitative labor, Rogers & Swinnerton (2008) argue that exploited child laborers are paid less than the value of their marginal product of labor, such that there is room for intervention in the exploitative sector that is welfare improving.}

As has been presented in previous literature, I show that economies exhibit child labor in the short run because of poverty. However, distinct from the literature, I bring in an explanatory factor for this reality and argue that the persistence of child labor over time is due to low school quality. In economies with high levels of school quality, the development path of the economy leads to an endogenous elimination of child labor relatively sooner without the need for policy intervention. An implication of this is that a policy that increases the quality of education in developing economies is superior to policies that deter child labor.

Historically, the successful stories of eliminating child labor come from the experience of developed countries, like Great Britain and the U.S. However, the evidence suggests that these results were not driven by legislative measures against child labor. For instance, Moehling (1999) and Kirby (2003) show that the U.S. and Britain’s measures against child labor including compulsory schooling, respectively, were enforced once the level of child labor...
was negligible, finding no statistically significant effects.\textsuperscript{4}

Interestingly, the quality of education in the U.S. a century ago was greater than the quality of education currently available in countries with child labor. Evidence of this can be found in the pupil-teacher ratio, a measure commonly used as a proxy for the quality of education. During the period when child labor fell in the U.S. (1880-1930), this country had an average ratio of 34.3 points while low income countries today have on average 50.2 points. As a reference, the ratio for the OECD countries today is 14.4 points.\textsuperscript{5} It is worth noticing that during this period the U.S. observed a relatively constant ratio.\textsuperscript{6}

Recent evidence is given in Figure 1. This Figure plots the correlation between child labor and quality of education across 82 countries. The horizontal axis denotes child labor, defined as the proportion of children between the ages 7-14 that work. The vertical axis denotes the quality of education, defined as the inverse of the student-teacher ratio. As the figure shows, across countries there is a negative correlation between child labor and the quality of education.

Empirical evidence of this relationship is presented in Handa (2002) who studies the importance of school quality on school attendance. The author argues that school enrolment in Mozambique is sensitive to the number of trained teachers. Moreover, building more schools or raising adult literacy has a larger impact on primary school enrolment rates than interventions that raise household income.

The rest of the paper is organized as follows. The next section presents the basic model of a small open economy, and section 3 characterizes the equilibrium. The analysis of the effects of a ban in child labor is presented in section 4. Section 5 considers the relationship between different economic institutions and the activity of the children. In particular, section

\textsuperscript{4}Moehling (1999) finds that the minimum age limits legislation and compulsory education had no effects on the likelihood of manufacturing child labor in the US: the decrease in child labor in US during 1880 – 1930 was not driven by the legislative success of the child labor movement. Kirby (2003) argues that the state legislation and education laws in Great Britain had no effect upon child labor. Moreover, by the time the child labor legislation and compulsory schooling were introduced, child employment had declined to statistically insignificant levels.


\textsuperscript{6}The ratios for the US are: 34.2 (1880), 35.0 (1890), 35.4 (1900), 34.4 (1910), 33.6 (1920), and 33.2 (1930). Actually, the pupil-teacher ratio remained roughly at the same levels until the 1960s.
5.1 examines the effect of the quality of education and property rights on the activity of the children; section 5.2 presents the case of imperfect credit markets; and section 5.3 discusses the effect of inequality. Section 6 presents the closed economy case and section 7 concludes. Proofs of all propositions are provided in the Appendix.

2 A model of child labor, crime and schooling

Consider an endogenous growth model of a small open overlapping-generations economy with no population growth. Every period, a continuum of agents with mass one is born. Each agent lives for three periods: childhood, adulthood and old age. All individuals can work and they are endowed with one unit of time each period. A household is defined as the parent-child pair. Following the unitary model of the household, parents are the decision makers of the family, and they exhibit altruistic behavior towards their children in the form of human capital bequest (investment in education). Each individual $i$ seeks to maximize

$$U = u(c_{ai}) + u(c'_{oi}) + u(l'_{oi}) + \delta v(h'_i) \equiv \ln (c_{ai}) + \ln (c'_{oi}) + \ln (l'_{oi}) + \delta \ln (h'_i) ,$$  

where $c_{ai}$ is the household $i$’s current consumption, $c'_{oi}$ is individual $i$’s consumption when old, $l'_{oi} \in [0,1]$ is individual $i$’s leisure when old, and $h'_i$ is the level of human capital of
individual $i$’s offspring once she is an adult. This last term represents the intergenerational altruism from the parent to the child. Primed variables denote next-period values.

Since children make no decisions, parents allocate their child’s unit of time between school, $e_i$, work, $x_i$, or illegal activities (crime), $z_i$, with

$$e_i + x_i + z_i \leq 1. \quad (2)$$

For simplicity, crime requires the full unit of the child’s time ($z_i = \{0, 1\}$), while schooling and child labor can be combined ($e_i, x_i \geq 0$).

All individuals $i$ born at any time $t$ inherit the level of human capital of their parents. The child’s accumulation of human capital requires time and resources devoted to education, and it is governed by a dynamic function of the form

$$h_i' = \left(1 + \theta e_i^b b_i^{1-\beta} - \epsilon z_i\right) h_i, \quad (3)$$

with $\beta \in (\frac{1}{2}, 1)$ and $\epsilon \in (0, \bar{\epsilon})$, where $\bar{\epsilon} < 1$ is given in the Appendix, $h_i$ is the stock of human capital of the household $i$; $e_i \in [0, 1]$ is the time the child of household $i$ spends in education; $z_i = \{0, 1\}$ is an indicator equal to one if the child is involved in crime, zero otherwise; $b_i \geq 0$ is the household $i$’s investment on the education of its child; and $\theta > 0$ is a parameter that reflects the quality of education in the economy.

The properties of the human capital accumulation function reflect some of the key elements in the related literature: (i) the individual’s level of human capital is an increasing function of the parental level of human capital, the time spent in education, and the resources invested in education; (ii) there are diminishing returns to the time and resources invested in education; and (iii) for a given level of parental human capital, time spent in education, and resources invested in education, the returns to education are higher the larger the quality of education.

From the above specification of human capital accumulation, note that a child devoting all of her time to school ($e_i = 1$) will have a level of human capital in the next period equal

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7See, for instance, Glomm and Ravikumar (1992) and Galor and Tsiddon (1997).
to \((1 + \theta b_1^{1-\beta})h_i\). In turn, a child spending all of her time working \((x_i = 1)\) will keep constant her level of human capital. At the other extreme, if a child engages in crime \((z_i = 1)\) her level of human capital in the next period is equal to \((1 - \varepsilon)h_i\). The assumption \(1 - \varepsilon > 0\) implies that even a full time criminal retains some human capital, and the assumption \(\varepsilon > 0\) implies that crime harms human capital accumulation more than work.

The technology of human capital accumulation is identical for all individuals (i.e. all individuals have the same ability). However, they may differ in their level of human capital inherited from their parents. In particular, I assume the economy starts at time 0 with a distribution of human capital of the initial parent generation given by the density function \(g_0(h_{i,0})\) with a continuous support defined over the nonnegative real line. It follows that

\[
G_t(m) = \int_0^m g_t(h_{i,t}) \, dh_{i,t} \quad \forall \ m \in \mathbb{R}_+ \quad \text{and} \quad \int_0^\infty g_t(h_{i,t}) \, dh_{i,t} = 1 \quad \forall \ t. \tag{4}
\]

Every period the economy produces a single final good using physical capital and the human capital of the working individuals. Given the small open economy assumption, the supply of physical capital every period is determined by the aggregate savings in the economy, in addition to net international borrowing. Factor and product markets are competitive. The final good is produced according to the production technology

\[
F(K, H) = AK^\alpha H^{1-\alpha}, \tag{5}
\]

with \(A > 0, \alpha \in (0, 1)\), where \(K\) is the aggregate stock of physical capital that fully depreciates every period, and \(H\) is the aggregate stock of effective human capital, with

\[
H \equiv H_a + \phi H_c + H_o = \int_0^1 h_i \, di + \phi \int_0^1 x_i h_i \, di + \int_0^1 (1 - \lambda_o) h_{o} \, di, \tag{6}
\]

where \(H_a\) is the aggregate stock of human capital provided by adults, \(H_c\) is the aggregate stock of human capital provided by children, \(H_o\) is the aggregate stock of human capital provided by old individuals, and \(\phi \in (0, 1]\) is the productivity of children relative to that of adults. Notice from equation (6) that I assume all worker individuals are perfect substitutes in production, reflecting the facts that a child works \(x_i \in [0, 1]\) units of time and they are
not more productive than adults, and the fact that an old individual works \(1 - l_{oi} \in (0, 1)\) units of time.

During childhood, individuals may receive schooling and/or go to work and receive a wage rate per effective unit of human capital \(w_c\), or they may be involved in crime appropriating crime rents. I assume that crime is a fully unproductive activity and only children can engage on it. If at least one household has its child involved in crime \((z_i = 1\) for some \(i\)), a fixed proportion \((1 - p)\) of total labor earnings in the economy is subject to appropriation, i.e. parents with children involved in crime are not exempt from appropriation. The parameter \(p \in (0, 1)\) reflects the security of effective property rights in the economy, with \(p = 1\) denoting perfectly secure property rights. Hence, a child involved in crime can claim a proportion \((1 - p) / n\) of the labor earnings \(Y_L\), where \(n\) is the total number of children involved in crime.

When adults, individuals supply their unit of labor inelastically in the labor market receiving a wage rate per effective unit of human capital \(w\), and decide the allocation of time of their child \((e_{i,t}, x_{i,t}, z_{i,t})\). In addition, from the household’s income, parents decide how much to invest in the education of their child \(b_i\), how much to save (or borrow) \(s_i\), and how much to consume \(c_{ai}\). In order to ensure boundedness on the long-run growth rate of the economy, I assume \(b_i \leq b_H\), for some \(b_H > 0\) to be defined below. Formally, household \(i\)’s consumption is given by

\[
c_{ai} = p(w + w_c x_i) h_i + \frac{(1 - p) Y_L}{n} z_i - s_i - b_i.
\]  (7)

When old, individuals decide the proportion of time they allocate into leisure and work receiving a wage rate \(w_o\), spending all the resources generated from net capital income and labor in their consumption. That is, whenever an adult borrow resources, she pays the debt when old. Thus, consumption for an old individual is

\[
c'_{aoi} = Rs_i + p (1 - l'_{ao}) w'_o h_{oi},
\]  (8)

where the gross interest rate, \(R = 1 + r\), is given by international capital markets and
assumed to be constant over time.\(^8\)

Every period, the aggregate resources lost to child crime add to aggregate crime rents. That is, the distribution of crime rents satisfy the aggregate consistency condition

\[
\int_0^1 (1 - p) (w + w_c x_i) h_i \, di + \int_0^1 (1 - p) (1 - l_{oi}) w_o h_{oi} \, di = (1 - p) \int_0^1 Y_L \frac{z_i}{n} \, di. \tag{9}
\]

In this economy, a competitive equilibrium is defined as a sequence of individual allocations \(\{e_i(t), x_i(t), z_i(t), b_i(t), l_{oi}(t + 1), s_i(t)\}_{t=0}^\infty\) for all agents \(i \in [0, 1]\), with \(s_i(-1) > 0\) given, a sequence of distributions of human capital \(\{G_t(h_i(t))\}_{t=0}^\infty\), with \(g_0(h_i(0))\) given, a sequence of allocations \(\{K(t)\}_{t=0}^\infty\), and a sequence of prices \(\{R(t), w(t), w_c(t), w_o(t)\}_{t=0}^\infty\) such that, given prices, each individual maximizes her utility, firms maximize profits, human capital for each individual evolves according to (3), with \(h_i(0) = h_{i0} > 0\) for all \(i \in [0, 1]\), the distribution of crime rents satisfies (9), and every market clears for all \(t \geq 0\).

### 3 Equilibrium analysis

Consider first the firms. Competitive international capital markets imply that the stock of physical capital in the economy at any time \(t\) is determined by

\[
\alpha \frac{F(K, H)}{K} = R, \tag{10}
\]

where \(R\) is constant and given in the international market.

Next, profit maximization implies that all units of human capital are paid according to their marginal product. Then, the wage per unit of human capital for adults, child labor, and old labor are, respectively, given by:

\[
w = (1 - \alpha) \frac{F(K, H)}{H}, \tag{11}
\]

\[
w_c = \phi w, \tag{12}
\]

\[
w_o = w. \tag{13}
\]

\(^8\)The small open economy assumption is relaxed in Section 6.
The small open economy assumption, by which the rental rate of physical capital is determined by the international market, simplifies the analysis by eliminating dynamics in the physical capital-effective human capital ratio of the economy. From equation (10), this ratio is stationary and given by

\[ K = \left( \frac{\alpha A}{R} \right)^{\frac{1}{1-\alpha}} H. \]  

(14)

In turn, this implies from equations (11)-(13) that the wage rates of the economy are constant over time, with

\[ w = (1 - \alpha) A^{1/(1-\alpha)} \left( \frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}}. \]  

(15)

Now consider the problem of the households. First, optimal saving satisfy the Euler equation

\[ \frac{\partial u (c_{ai})}{\partial c_{ai}} = R \frac{\partial u (c'_{ai})}{\partial c'_{ai}}, \]  

(16)

for all \( i \), which equates the marginal rate of substitution between current and future consumption of an adult with the (constant) marginal rate of transformation.

Second, the optimal choice of leisure is always interior and it satisfies

\[ \frac{\partial u (c'_{ai})}{\partial c'_{ai}} \frac{\partial c'_{ai}}{\partial l'_{oi}} + \frac{\partial u (l'_{oi})}{\partial l'_{oi}} = 0, \]  

(17)

for all \( i \), where the marginal benefits come from the utility leisure gives to an individual, and the marginal costs come from the reduction in future consumption associated with lower labor income, with

\[ \frac{\partial c'_{ai}}{\partial l'_{oi}} = -p w'_o h_i. \]

Using the fact that \( w_c = \phi w \), one can easily verify that the system given by equations
and (17) results in the following general expressions for savings and leisure for all $i$:

$$s_i = \frac{2}{3} pwh_i \left[ 1 + \phi (1 - e_i) + \frac{(1 - p) Y_L}{npwh_i} z_i - \frac{b_i}{pwh_i} - \frac{1}{2R} \right],$$

(18)

$$l'_{oi} = \frac{1}{2} + \frac{Rs_i}{2pwh_i}.$$  (19)

Consequently, from equations (7) and (18), consumption when adult is given by

$$c_{ai} = \frac{1}{3} pwh_i \left[ 1 + \phi (1 - e_i) + \frac{(1 - p) Y_L}{npwh_i} z_i - \frac{b_i}{pwh_i} + \frac{1}{R} \right],$$

(20)

and from the Euler equation in (16), consumption when old satisfies $c'_{oi} = Rc_{ai}$.

Third, the optimal allocation of schooling satisfies.

$$\frac{\partial u(c_{ai})}{\partial c_{ai}} \frac{\partial c_{ai}}{\partial x_i} \frac{\partial x_i}{\partial e_i} + \delta \frac{\partial v(h'_0)}{\partial h'_i} \frac{\partial h'_i}{\partial e_i} \leq 0,$$

(21)

with equality whenever optimal schooling is interior. The marginal benefits come from the increase in the child’s future human capital associated with schooling

$$\frac{\partial h'_i}{\partial e_i} = \beta \theta \left( \frac{b_i}{e_i} \right)^{1-\beta} h_i,$$

which are increasing on the quality of education, $\theta$. The marginal costs come from the reduction in the household’s current consumption associated with lower child labor income

$$\frac{\partial c_{ai}}{\partial x_i} \frac{\partial x_i}{\partial e_i} = -pwch_i.$$

Next, the optimal choice of investment in education $b_i$ satisfies

$$\frac{\partial u(c_{ai})}{\partial c_{ai}} \frac{\partial c_{ai}}{\partial b_i} + \delta \frac{\partial v(h'_0)}{\partial h'_i} \frac{\partial h'_i}{\partial b_i} \leq 0,$$

(22)

with equality whenever the optimal investment in education is interior. The marginal benefits come from the increase in the child’s future human capital

$$\frac{\partial h'_i}{\partial b_i} = (1 - \beta) \theta \left( \frac{e_i}{b_i} \right)^\beta h_i,$$
while the marginal costs come from the reduction in the household’s current consumption associated with the investment on education

\[
\frac{\partial c_{ai}}{\partial b_i} = -1.
\]

The equality of the returns to schooling and the returns for the investment on education given in equations (21) and (22), respectively, give the following relationship between schooling and investment on education

\[
b_i = \frac{1 - \beta}{\beta} pwh_i e_i.
\] (23)

Thus, the equality of returns to both activities implies that the investment on education is increasing in both, the optimal allocation of schooling and the level of human capital of the household.

The condition for the optimality of schooling in equation (21) together with the equality of the returns to schooling and the investment on education in equation (23) determine the optimal level of schooling at time \( t \), which is given by

\[
e_i = \frac{3}{3 + \delta} \left[ \frac{\delta \beta}{3 \phi} \left( 1 + \phi + \frac{1}{R} \right) - \frac{1}{\theta} \left( \frac{\beta}{(1 - \beta) pwh_i} \right)^{1-\beta} \right], \quad (24)
\]

with \( \partial e_i/\partial h_i > 0 \) and \( \partial e_i/\partial \theta > 0 \). Together, equations (23) and (24) imply that both, the optimal choice of schooling and investment on education are increasing in the level of human capital of the household and the quality of education.

Notice from equation (24) that we have \( e_i = 0 \) if and only if \( h_i \leq h_L \), with

\[
h_L = \frac{\beta}{(1 - \beta) pwh} \left[ \frac{3 \phi}{\theta \delta \beta \left( 1 + \phi + \frac{1}{R} \right)} \right]^{\frac{1}{1-\beta}}, \quad (25)
\]

where \( w \) is given by equation (15). Similarly, we have that \( e_i = 1 \) if and only if \( h_i \geq h_H \), with

\[
h_H = \frac{\beta}{(1 - \beta) pwh} \left[ \frac{3 \phi}{\theta \left[ \delta \beta \left( 1 + \phi + \frac{1}{R} \right) - (3 + \delta) \phi \right]} \right]^{\frac{1}{1-\beta}}, \quad (26)
\]
where the right-hand-side of this expression is defined if and only if $\delta > \delta_L$, with

$$\delta_L \equiv \frac{3\phi}{\beta (1 + \phi + \frac{1}{R})} - \phi,$$

where $\delta_L > 0$ since $\beta > 1/2$. Thus, whenever $h_H$ exists, i.e. $\delta > \delta_L$, we have $h_H > h_L$.

It is worth noticing that, since $e_i \leq 1$, the left-hand-side of equation (21) is positive whenever $h_i > h_H$. In this case, equation (23) does not hold. Instead, from equation (22) one can verify that the optimal investment on education is given by the unique solution to the following expression:

$$\frac{3}{\theta} b_i^\beta + (3 + \delta (1 - \beta)) b_i = \delta (1 - \beta) \left(1 - \frac{1}{2R}\right) pwh_i,$$

(27)

with $db_i/dh_i > 0$ and $db_i/d\theta > 0$.

Finally, in order to analyze the relationship between child labor, child crime and human capital, recall that the crime rents for a household with its child involved in crime are given by $(1 - p) Y_L/n$, where $Y_L = (1 - \alpha) Y$. Given that crime involves the whole unit of time of the child, for child crime to be optimal a necessary condition is that the crime rents must be greater than the full-time child labor earnings. However, it is not a sufficient condition since crime harms the accumulation of human capital of the child more than full time child labor. Therefore, child crime is optimal if and only if the household’s utility from child crime is greater or equal than the utility from full time child labor. Using equations (7), (8), (18), and (19), one can verify that $z_i = 1$ if and only if

$$(1 - \varepsilon)^{\delta/3} \left(1 + \frac{(1 - p)(1 - \alpha) Y}{npwh_i} + \frac{1}{R}\right) \geq 1 + \phi + \frac{1}{R}.$$  

(28)

Substituting equation (11) into this expression, and rearranging terms, we have $z_i = 1$ if and only if $h_i \leq h_c$, where $h_c$ is given by the unique solution to the following expression

$$h_c G(h_c) = \frac{(1 - \varepsilon)^{\delta/3} (1 - p) H}{p \left[\phi + (1 + \frac{1}{R}) \left(1 - (1 - \varepsilon)^{\delta/3}\right)\right]},$$

(29)

where I used the fact that $n = G(h_c) = \int_0^{h_c} g(h_i) dh_i$, and $G(h_c) \in (0, 1)$ is the proportion
of the households with a level of human capital up to $h_c$. Thus, $G(h_c)$ is the proportion of children involved in crime. Notice from this equation that the threshold $h_c$ increases over time whenever the aggregate stock of effective human capital, $H$, increases over time.

We have then the following

**Proposition 1** (i) There exists a threshold level of human capital $h_c > 0$, such that $z_i = 1$ if and only if $h_i \leq h_c$. (ii) There exists a threshold level of human capital $h_L > 0$ with $h_L > h_c$ if and only if $p \geq p_L$, with $p_L < 1$, such that $x_i = 1$ if and only if $h_i \in (h_c, h_L)$. (iii) There exists a threshold level of human capital $h_H > 0$ if and only if $\delta > \delta_L$, with $h_H > h_L$, such that $e_i = 1$ if and only if $h_i \geq h_H$. (iv) If $h_i \in (h_L, h_H)$, then $x_i > 0, e_i > 0$ with $x_i + e_i = 1$ and $\partial e_i / \partial h_i > 0$, where $p_L$ is given in the Appendix.

Part (i) of the Proposition says that the children of those households with the lowest levels of human capital in the economy ($h_i \leq h_c$) are the children involved in crime. Part (ii) provides the condition for full time child labor to exist. It says that there exists a non-empty interval of human capital ($h_c, h_L$) if and only if the security of effective property rights is sufficiently large, such that the children of those households with levels of human capital within this interval work full time. Otherwise, the rents from child crime are sufficiently large such that households have no incentives for full-time child labor, even though child crime harms the accumulation of human capital of the child more than full-time child labor.

Part (iii) provides the condition for full-time schooling to exist. It says that the children of those households with the largest levels of human capital $h_i \geq h_H$ attend school full-time, where $h_H$ exists if and only if the degree of altruism of the parent to her child, $\delta$, is sufficiently large. Part (iv) says that the children of those households with intermediate levels of human capital, $h_i \in (h_L, h_H)$, combine both child labor and schooling, and that the level of schooling is increasing in the level of human capital of the household.

Intuitively, households with relatively large levels of human capital ($h_i \geq h_H$) receive a sufficiently high income from adult labor such that they do not need additional income from child labor, sending their children to school full time. On the opposite side of the distribution, for those households with relatively low levels of human capital ($h_i \leq h_L$), the
labor income parents receive is sufficiently low such that their children do not attend school. Instead, the children participate in either full-time child labor or child crime in order to obtain the resources needed for the consumption of the household.

In what follows I assume \( p > p_L \) and \( \delta > \delta_L \), such that both full-time child labor and full-time schooling exist in equilibrium.

Consequently, from equation (18) we have four possible cases for optimal savings depending on the level of human capital of the household. The first case is given by those households with their children participating in crime. That is, for all \( h_i \leq h_c \)

\[
s_i = \frac{2}{3} pwh_i \left[ 1 + \frac{(1 - p) H}{G(h_c) ph_i} - \frac{1}{2R} \right],
\]

(30)

where \( s_i > 0 \) for all \( h_i < h_c \), with \( \partial s_i / \partial h_i > 0 \), and \( w \) is constant and given by equation (15). In the second case we have those households with their children working full time, where for all \( h_i \in (h_c, h_L] \)

\[
s_i = \frac{2}{3} pwh_i \left[ 1 + \phi - \frac{1}{2R} \right],
\]

(31)

where \( s_i > 0 \) for all \( h_i \in (h_c, h_L] \), with \( \partial s_i / \partial h_i > 0 \). Next, we have those households with their children combining school and work. That is, for all \( h_i \in (h_L, h_H) \)

\[
s_i = \frac{2}{3} pwh_i \left[ 1 + \phi \left( 1 - \frac{e_i}{\beta} \right) - \frac{1}{2R} \right].
\]

(32)

By substituting equation (24) into equation (32), one can verify that there exists a level \( \tilde{h} \in (h_L, h_H) \) such that \( \partial s_i / \partial h_i \leq 0 \) if and only if \( h_i \geq \tilde{h} \). Furthermore, \( s_i < 0 \) at \( e_i = 1 \) if and only if \( \phi > \frac{\beta}{1-\beta} \left( 1 - \frac{1}{2R} \right) \equiv \tilde{\phi} \), where \( \tilde{\phi} < 1 \) if and only if \( \beta < 1/ \left( 2 - \frac{1}{2R} \right) \equiv \tilde{\beta} \in (1/2, 1) \).

Finally, the last case is given by those households with their children attending school full time, where for all \( h_i \geq h_H \) we have

\[
s_i = \begin{cases} 
\frac{2}{3} pwh_i \left[ 1 - \frac{1}{2R} - \frac{b_i}{pwh_i} \right] & \text{if } b_i < b_H \\
\frac{2}{3} pwh_i \left[ 1 - \frac{1}{2R} - \frac{b_H}{pwh_i} \right] & \text{if } b_i \geq b_H
\end{cases}
\]

(33)

where \( b_i \) in this last equation is given by the unique solution to equation (27). To simplify,
it is assumed that $b_H$ satisfies

$$b_H = \frac{1 - \beta}{\beta} \phi p (1 - \alpha) A^{1/(1-\alpha)} \left( \frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} h_H,$$

where $h_H$ is given by equation (26). This assumption implies $b_i = b_H$ for all $h_i \geq h_H$, and $b_i < b_H$ for all $h_i < h_H$ (see equation (23)). That is, once the child of a household attends school full time, the investment on education is constant. Thus, equation (33) becomes

$$s_i = \frac{2}{3} p wh_i \left[ 1 - \frac{1}{2R} - \frac{\phi (1 - \beta) h_H}{\beta h_i} \right].$$

(35)

Graphically, the optimal household’s saving as a function of the human capital of the household is given in Figure 2.

Figure 2: Household’s saving

Let us now analyze the evolution over time of the distribution of human capital in the economy. In order to do so, let $\bar{h}_{t,[a,b]}$ denote the average level of human capital in the interval $[a, b]$ at time $t$, and $\tilde{h}_{t,[a,b]}$ denote the $g_t(h)$-weighted average of $h_{i,t}$ in the interval
Thus,  

$$\overline{h}_{t,[a,b]} = \int_a^b h_{i,t} g_t(h_{i,t}) \, dh_{i,t} = \int_a^b g_t(h_{i,t}) \, dh_{i,t},$$

where the last equality follows from the Mean Value Theorem since $g_t(h_{i,t}) \geq 0$ for any $h_{i,t} \in \mathbb{R}$. 

Let $\Delta B \equiv B_{t+1} - B_t$ denote the change over time of the variable $B$. Recalling that the level of human capital of a household with a child working full time remains constant, the aggregate level of human capital in the economy, $h = \int_0^1 h_i \, di$, evolves according to

$$\Delta h_{t,[0,\infty]} = \Delta \tilde{h}_{t,[0,h_c]} G(h_c) + \Delta \tilde{h}_{t,[h_c,h_H]} [G(h_H) - G(h_L)] + \Delta \tilde{h}_{t,[h_H,\infty]} [1 - G(h_H)],$$

where $\Delta \tilde{h}_{t,[0,h_c]} < 0$, $\Delta \tilde{h}_{t,[h_c,h_H]} > 0$, and $\Delta \tilde{h}_{t,[h_H,\infty]} > 0$. One can easily verify that $\partial \Delta h_{t,[0,\infty]} / \partial \theta > 0$, that is, the evolution of the level of human capital in the economy is increasing in the quality of education.

Notice from equation (3) that, at any time $t$, for all the children living in households with a level of human capital $h_i \leq h_c$, since $z_i = 1$, we have $h_i' / h_i < 1$. In turn, this implies from equation (29) that once in adulthood, children living in households with human capital levels within this interval will have their own children engaged in child crime. This implies that, in the long-run, $\lim_{T \to \infty} h_{t,[h_c,\infty]} = 0$.

For the children working full time, i.e. $h_i \in (h_L, h_c]$, since $x_i = 1$ and $z_i = e_i = 0$, their human capital next period is constant ($h_i' = h_i$). However, note from equations (25) and (29) that $h_L$ is constant and $\Delta h_c > 0$. Therefore, there exists some period $T < \infty$ with $\partial T / \partial \theta < 0$ in which $h_c = h_L$, such that the dynasties of those households that in the beginning had their children working full time, will have their children engaged in child crime. This implies that, in the long-run, $\lim_{t \to \infty} \Delta h_{t,[h_L,\infty]} = 0$.

Next, for all $h_i \in (h_L, h_H)$ we have $h_i' > h_i$ since $e_i \in (0,1)$ and $b_i > 0$. Moreover, given that both $e_i$ and $b_i$ are increasing in $h_i$, every generation accumulates more human capital and will increase their adult labor earnings compared to the previous generation, choosing in turn to send their own child more time to school and investing additional resources on her education. Following this process, there exists some period $T' < \infty$ with $\partial T'/\partial \theta < 0$
in which the dynasties of the households within this segment of the distribution will choose full-time schooling.

Thus, the development path of this economy leads to an endogenous elimination of child labor in the long-run. More importantly, the facts that \( \partial T/\partial \theta < 0 \) and \( \partial T'/\partial \theta < 0 \) imply that the persistence of child labor in an economy is lower the larger the quality of education.

In the steady state equilibrium, the distribution of human capital is characterized by a unique sequence of optimal choices for the child’s time allocation: a constant proportion \( G(h^*_L) \) of dynasties have their children involved in crime, and a constant proportion \( 1 - G(h^*_L) \) have their children attending school full-time, where \( h^*_L = h_L \). Thus, in the long run, the growth rate of the aggregate level of human capital in the economy converges asymptotically to

\[
\lim_{t \to \infty} \frac{\Delta h_t_{[0,h_L]}}{h_t_{[0,h_L]}} = \theta b_H^{1-\beta}.
\]

Similarly, from equations (6) and (36), we have that the evolution over time of the aggregate stock of effective human capital, \( H_t \), is given by

\[
\Delta H_t = \Delta h_t_{[0,\infty]} + \phi \Delta \alpha_t h_t_{[h_L,h_H]} [G(h_H) - G(h_L)] + \Delta(1 - I_{ot}) h_t_{[0,\infty]}.
\]

Noticing that leisure is constant in the long run, then the growth rate of the economy converges asymptotically to

\[
\lim_{t \to \infty} \frac{\Delta Y_t}{Y_t} = \theta b_H^{1-\beta},
\]

where \( \frac{\Delta Y_t}{Y_t} = \frac{\Delta h_t}{H_t} = \frac{\Delta K_t}{K_t} \) for all \( t \geq 0 \).

To provide further intuition regarding the evolution of the children’s activities, Figure 3 illustrates in a simple manner the transitional dynamics of the allocations of children’s time. The vertical axis denotes the mass of children, i.e., a point in the vertical axis represents the child in household \( i \in [0,1] \). For expositional reasons, households are conveniently indexed so that children are ranked in order of increasing human capital. That is, the family at the bottom of the graph has the lowest level of human capital in the economy at time 0, and the family at the top of the graph has the highest level of human capital. The horizontal axis denotes time. Thus, along any horizontal line we are following the dynasty of the same
Figure 3: Allocations of children’s time across dynasties over time

Figure 3 shows two examples. Case (a) follows the dynasty of a household that at time 0 has its children working part-time, and after some generations, the child of the corresponding dynasty attends school full time. The reason is that the level of human capital of this dynasty increases over time. Then, the household’s adult labor income increases as well such that at some point it is optimal for this dynasty to send its child full-time to school. Case (b) follows a dynasty with a lower level of human capital that at time 0 has its child working full-time, and after some generations the child is engaged in crime. The reason is that, as the economy develops, the rents from child crime increase over time, increasing in turn the relative earnings between child crime and child labor. Thus, the rise in the relative earnings induces this dynasty to reallocate its child’s time from child labor to child crime in the future.

4 Analysis of a child labor ban

Formally, consider an enforceable full-ban on child labor occurring at some period $t_b > 0$, such that $x_i = 0$ for all $i$. After the ban is imposed, it is optimal for a household to have its child in school full time if and only if the utility from schooling is greater than the utility from child crime. In the Appendix, I show that there exists a threshold level $h'_c \in (h_c, h_H)$ such that, after the ban, a household will have its child attending school if and only if $h_i > h'_c$. Furthermore, $h'_c > h_L$ if and only if $p < p_H$ where $p_H > p_L$. That is, the new threshold
level of human capital that determines whether a household prefers schooling over child crime after the ban, $h'_L$, is greater than the threshold level of human capital that determines whether a household starts sending its child to school part time before the ban, $h_L$, if and only if the level of property rights is sufficiently low. We have then the following

**Proposition 2** (i) A permanent ban on child labor increases the proportion of children involved in crime in the long run if and only if $p < p_H$ and $\theta < \theta_H$. (ii) A temporary ban on child labor increases the proportion of children involved in crime in the long run if and only if $p < p_H$ and $\theta < \theta_L$, with $\theta_L > 0$ and $\theta_L < \theta_H$. (iii) Both $\theta_L$ and $\theta_H$ are decreasing in $t_b$. (iv) If $p < p_H$ and $\theta < \theta_L$, even a temporary ban has permanent negative effects for the children living in the poorest households.

Parts (i) and (ii) of the Proposition state that both a permanent ban and a temporary ban on child labor will permanently increase the proportion of children engaged in crime if and only if the security of effective property rights and the quality of education in the economy are sufficiently low. Part (iii) of the Proposition implies that, the earlier a ban on child labor is implemented, the more likely is that the ban will increase the proportion of children in crime.

Figure 4 illustrates in a simple manner the intuition behind this result. The figure follows the same household $i$, and its dynasty, that at time $t = 0$ has a level of human capital just above the level $h_L$, and at the time when the ban is implemented, $t_b$, it has a level of human capital $h_i (\theta)$. Figure 4 shows the case $p < p_H$ such that $h'_L > h_L$. Panels (a) and (b) depict the conditions for a permanent ban and a temporary ban to increase child crime in the long run, respectively.

Recall from the analysis of the previous section that the proportion of households with their children involved in crime in the long run is given by $G(h_L)$. Thus, a necessary condition for a permanent child labor ban to increase the proportion of children involved in crime in the long run is $p < p_H$, such that $h'_L > h_L$. However, this is not a sufficient condition since a child combining school and work before the ban, i.e. $h_i \in (h_L, h_H)$, will be involved in crime once the ban is implemented if and only if $h_i \leq h'_L$. Together, $p < p_H$ and $\theta < \theta_H$ are
necessary and sufficient conditions for a ban to increase child crime in the long run.

To see this, consider panel (a) in the figure. Everything else equal, this panel presents two cases for $p < p_H$: case 1, in which the quality of education is relatively large such that $\theta_1 > \theta_H$; and case 2, in which the quality of education is relatively low such that $\theta_2 < \theta_H$.

In case 1, the quality of education is sufficiently large such that at the time the policy is implemented, there are no households with a level of human capital between $h_L$ and $h_c^*$ (point A in panel (a)). In this case, the permanent ban has no long run effects. However, if the quality of education is sufficiently low as in case 2 ($\theta_2 < \theta_H$), the accumulation of human capital of the children that attend school part time is not sufficiently large, such that, at the time the ban is implemented, there exists a non-empty set of dynasties with $h_i \in (h_L, h_c^*)$ (point B in panel (a)). In this case, the permanent ban on child labor increases child crime in the long run since $G(h_c^*) > G(h_c^*)$. Finally, notice from the figure that, for a given $\theta$, the earlier a ban is implemented (lower $t_b$), the more likely the policy will increase child crime in the long run.

Consider now the case of a temporary ban on child labor (panel (b) in Figure 4). For simplicity, suppose a ban on child labor is implemented at time $t_b$ and it is abrogated at time $t_{b+1}$. Notice that once the policy is abrogated, the threshold level $h_L$ applies again. In this case, since crime harms the human capital accumulation of the child, if the quality of education in the economy is sufficiently low such that $\theta < \theta_L$ (case 3 in panel (b)), there exists a non-empty set of dynasties negatively affected by the policy that before the
policy implementation would have had their children in school in the long-run, but after the temporary implementation will have their children involved in crime in the long-run. Therefore, if the quality of education is sufficiently low such that $\theta < \theta_L$, the long-run proportion of children in crime after the temporary ban on child labor is greater relative to the same proportion without the policy intervention, i.e. $G(h^T_c) > G(h^*_c)$.

Finally, let us analyze the effects of the child labor ban on output and welfare in the long run. Notice that, at the period the ban is implemented ($t_b$), the aggregate stock of effective human capital in the economy ($H$) decreases since there are no working children anymore. Next, given the small open economy assumption, notice from equation (14) that the aggregate stock of physical capital decreases to such a level that the ratio $K/H$ remains constant. From equation (11), this implies that the wage per unit of human capital for adults is not affected by the policy. In turn, this implies that in the long run, the dynasties of all those households with a level of human capital $h_i > h'_e$ at the time of the policy implementation, are not affected by the child labor ban.

Either in the case of a permanent ban or a temporary ban on child labor, if the quality of education is sufficiently low such that any policy increases the level of crime in the long run (i.e. $G(h^P_c) > G(h^*_c)$), the aggregate stock of effective human capital after the policy ($H^P_t$) is lower relative to the no-ban case ($H_t$) every period (i.e. $H^P_t < H_t$ for all $t \geq t_b$), which implies that both physical capital and output are also lower every period because of the ban ($K^P_t < K_t$ and $Y^P_t < Y_t$ for all $t \geq t_b$). Furthermore, the fact that lower output induces lower aggregate labor income, together with the fact that $G(h^P_c) > G(h^*_c)$, imply that crime rents per household decrease. Thus, the dynasties of all those households with $h_i \leq h'_e$ are worse off because of the ban.

5 Institutions and the activity of children

In this section, I use the above model to examine how the children’s activities are affected by the institutional quality on the economy, by the presence of credit market imperfections, and finally I briefly discuss the effects of inequality.
5.1 Property rights and the quality of education

In this section, we examine the relationship between the institutional quality in the economy and the incidence of child labor and child crime. In particular, the focus is on the quality of education ($\theta$) and the security of effective property rights ($p$). Historically, the successful stories of eliminating child labor come from the experience of developed countries, like Great Britain and the U.S. Interestingly, the quality of education in the U.S. a century ago was greater than the quality of education currently observed in developing countries with child labor. Accordingly, from equations (25), (26), and (29), we have the following result:

**Proposition 3** An exogenous increase in

(i) the quality of education: increases the proportion of children attending school full time and decreases the proportion of children working full time. Furthermore, it reduces both the time it takes to eliminate child labor from the economy and the level of child crime in the long run;

(ii) the security of effective property rights: increases the proportion of children attending school full time, decreases the proportion of children out from school, and decreases the level of child crime in the short and the long run.

This result suggests that countries with higher institutional quality will observe relatively larger levels of children attending school, and relatively lower levels of children outside from school. The reason is simple, the greater the quality of institutions, the greater the effective returns to education, whether they come from increasing future human capital ($\theta$), or from increasing future net labor income ($p$).

5.1.1 Investment on institutional quality

We now explore the case where the institutional quality of an economy can be improved by investing resources on it. The purpose of this section is to present a simple analysis of the effects that the investment on either school quality or the security of property rights can have on the children’s activities. An analysis of optimal investment on institutional quality is beyond the scope of the paper.
Suppose that every period $t$ the levels of quality of education and effective property rights are determined by

$$
\theta = \Psi_\theta (I_\theta), \\
p = \Psi_p (I_p),
$$

where $I_j$ denotes the aggregate resources invested on institution $j$, with $\partial \Psi_j / \partial I_j > 0$ for $j = \{\theta, p\}$, $\Psi_\theta (0) = \theta_0 > 0$, $\Psi_p (0) = p_0 \in (0, 1)$, $\lim_{I_\theta \to \infty} \Psi_\theta (I_\theta) = \bar{\theta}$, and $\lim_{I_p \to \infty} \Psi_p (I_p) = 1$.

In addition, suppose the investment on institutions is financed through taxes. In particular, assume that every period $t$ each adult $i$ faces a tax $T_{\theta i}$ to finance the investment on the quality of education, and a tax $T_{p i}$ to finance the investment on the security of property rights. Accordingly, adult consumption is now given by

$$
c_{ai} = pwh_i + px_iw_c h_i + \frac{(1 - p) Y_L}{n} z_i - s_i - b_i - T_{\theta i} - T_{p i}. 
$$

Every period, aggregate investment on an institution is given by the aggregate tax revenue associated with it,

$$
I_\theta = \int_0^1 T_{\theta i} di, \\
I_p = \int_0^1 T_{p i} di.
$$

To maintain the analysis as simple as possible, assume $T_{ji}$ is proportional to the adult labor income of the household, such that $T_{\theta i} = \tau_\theta pwh_i$ and $T_{p i} = \tau_p pwh_i$, where $\tau_j$ is the tax rate. Thus, $I_j = \tau_j pwh$ for $j = \{\theta, p\}$, where $h = \int_0^1 h_i di$.

It is straightforward to verify that the optimal level of schooling at time $t$ is now

$$
e_i = \frac{3}{3 + \delta} \left[ \frac{\delta \beta}{3 \phi} \left( 1 + \phi + \frac{1}{R} - \tau_\theta - \tau_p \right) - \frac{1}{\bar{\theta}} \left( \frac{\beta}{(1 - \beta) \phi pwh_i} \right)^{1-\beta} \right]. 
$$

Following the same steps as before, we have that $e_i = 0$ if and only if $h_i \leq h_L$, with

$$
h_L = \frac{\beta}{(1 - \beta) \phi pw} \left[ \frac{3 \phi}{\theta \delta \beta \left( 1 + \phi + \frac{1}{R} - \tau_\theta - \tau_p \right) \bar{\theta}} \right]^\frac{1}{1 - \beta},
$$
where \( w, \theta \) and \( p \) are given by equations (11) and (39), respectively. Next, we have that 
\[ e_i = 1 \text{ if and only if } h_i \geq h_H, \]
with
\[
h_H = \frac{\beta}{(1 - \beta) \phi pw} \left[ \theta \left[ \delta \beta \left( 1 + \phi + \frac{1}{R} - \tau_{\theta} - \tau_p \right) - (3 + \delta) \phi \right] \right]^\frac{1}{1 - \beta}. \tag{44}
\]

Similarly, one can easily verify that \( z_i = 1 \) if and only if \( h_i \leq h_c \), where \( h_c \) is given by the unique solution to the following expression
\[
h_c G (h_c) = \frac{(1 - \varepsilon)^{\delta/3} (1 - p) H}{p \left[ \phi + (1 + \frac{1}{R} - \tau_{\theta} - \tau_p) \left( 1 - (1 - \varepsilon)^{\delta/3} \right) \right]}, \tag{45}
\]
where \( G (h_c) \in (0, 1) \) is the proportion of child crime in the economy.

Next, we examine how the two alternative policies affect the activity of children. As indicated previously, a child’s activity depends on the level of human capital of the household relative to these thresholds’ levels. Thus, whether a child switch between activities or not depends on the effect the investment on institutional quality has on the different thresholds.

Consider first the case of the investment on the quality of education, \( \theta \). Given equations (39) and (41), total differentiation of equation (43) with respect to \( h_L, \theta, \) and \( \tau_{\theta} \) yields
\[
dh_L < 0 \iff \frac{\partial \Psi_{\theta} (I_{\theta})}{\partial I_{\theta}} \Psi_{\theta} (I_{\theta}) > \frac{1}{pwh \left( 1 + \phi + \frac{1}{R} - \tau_{\theta} \right)}. \tag{46}
\]
That is, the threshold \( h_L \) decreases with the investment on the quality of education if and only if the proportional increase on the school quality is sufficiently large. This implies that, if this proportional increase in \( \theta \) is sufficiently large, there exists a non-empty interval of the level of human capital such that, due to the investment, the children of the households with a human capital level within this interval will attend school part time when otherwise they would have worked full time. However, it is the case that some children would now work full time if the increase in \( \theta \) is not sufficiently large.

Similarly, from equation (44) we have that the threshold \( h_H \) decreases with the investment on the quality of education if and only if the proportional increase on \( \theta \) is sufficiently large.
That is,
\[
dh_H < 0 \iff \frac{\partial \Psi_\theta (I_\theta) / \partial I_\theta}{\Psi_\theta (I_\theta)} > \frac{1}{pwh \left(1 + \phi + \frac{1}{R} - \tau_\theta - \left(\frac{3+\delta}{\delta^2}\right) \phi \right)}.
\] (47)

Notice from equations (46) and (47), the increase in \( \theta \) needed to decrease \( h_H \) is relatively larger than the increase needed to decrease \( h_L \).

Thus, if the increase in \( \theta \) is sufficiently low, such that the inequality in equation (46) is reversed, the policy designed to increase the quality of education in the economy ends up driving children out from school. The reason is that, on the one hand, an increase in the quality of education gives incentives to increase the allocation of the child’s time in school. On the other hand, the fact that all households have to pay for the investment on the school quality gives incentives to households to allocate their child’s time away from school in order to obtain resources for the household. Overall, if the increase in \( \theta \) does not compensate for the resources lost with the investment, households will prefer to decrease the allocation of time in school.

Next, notice from equation (45) that the threshold \( h_c \) necessarily increases with the investment on school quality. Then, because of the policy, the children of some households will participate in child crime when otherwise they would have worked full time. The reason is that the tradeoff the poorest households face when deciding whether the child will work full time or will be engaged in child crime is independent of \( \theta \), while at the same time, these households spend resources on the tax.

Consider now the case of the investment on the security of effective property rights, \( p \). Following the same steps as before, from equations (43) to (45) we have

\[
dh_L < 0 \iff \frac{\partial \Psi_p (I_p) / \partial I_p}{\Psi_p (I_p)} > \frac{1}{(1 - \beta) pwh \left(1 + \phi + \frac{1}{R} - \tau_p \right)}.
\] (48)

\[
dh_H < 0 \iff \frac{\partial \Psi_p (I_p) / \partial I_p}{\Psi_p (I_p)} > \frac{1}{(1 - \beta) pwh \left(1 + \phi + \frac{1}{R} - \tau_p - \left(\frac{3+\delta}{\delta^2}\right) \phi \right)}.
\] (49)

\[
dh_c < 0 \iff \frac{\partial \Psi_p (I_p) / \partial I_p}{\Psi_p (I_p)} > \frac{1 - p}{pwh \left(1 + \frac{\phi}{1-(1-\delta)^{\gamma/\beta}} + \frac{1}{R} - \tau_p \right)}.
\] (50)
Similarly, we have from equations (48) and (49) that both thresholds $h_L$ and $h_H$ decrease with the policy if and only if the proportional increase in the level of effective property rights is sufficiently large. In this case, however, the threshold for the crime-work tradeoff, $h_c$, can indeed decrease with the policy if and only if the proportional increase in $p$ is sufficiently large, such that the inequality in equation (50) holds. Otherwise, even an investment policy that increases the security of property rights in the economy can drive children into crime.

Finally, it is worth noticing that, if we have access to differentiated tax rates across households, i.e. $\tau_{\theta_i}$ and $\tau_{p_i}$, we can implement a tax policy that necessarily decreases the different threshold levels, i.e. that increases schooling and decreases crime. For instance, suppose $\tau_{\theta_i} = \tau_{p_i} = 0$ for all households with $h_i < \tilde{h}$, for some $\tilde{h} > h_H$, and $\tau_{\theta_i} > 0$, $\tau_{p_i} > 0$ for all households with $h_i \geq \tilde{h}$. In this case, the relatively poor households would not incur in any cost associated with the investment in institutional quality, and they would be favored by the improvement on institutions. However, this would be at the expense of the relatively wealthy households.

### 5.2 Credit market imperfections

In this section we examine how credit market imperfections may affect the optimal decisions over the children’s activities. In particular, I focus on the case where credit market imperfections prevent households from borrowing resources. Formally, consider that household’s saving is restricted by a non-positive lower bound. For simplicity, assume that households cannot borrow more than a proportion $\pi \geq 0$ of the household’s adult labor income, such that $s_i \geq -\pi pwh_i$. Larger values of $\pi$ correspond to weaker credit market imperfections, and if $\pi = 0$, households simply cannot borrow. Otherwise, the model is as before.

Let $\lambda_i \geq 0$ denote the shadow value associated with the borrowing constraint $s_i \geq -\pi pwh_i$. Given equations (3), (7) and (8), the Lagrangian associated with household $i$’s maximization problem is

$$\mathcal{L}(s_i, l_a^i, e_i, h_i, z_i) = \ln (c_{a,i}) + \ln (c_{e,i}) + \ln (l_a^i) + \delta \ln (h_i^i) + \lambda_i (s_i - \pi pwh_i), \quad (51)$$

where $\lambda_i > 0$ whenever the restriction binds. Clearly, whenever $\lambda_i = 0$, the optimal alloca-
tions are the same as before. It is worth noticing that $\phi > \hat{\phi}$ is a necessary condition for $\lambda_i > 0$ for some $i$, since otherwise optimal saving is positive for all households (see Figure 2). In addition, notice that $\lambda_i = 0$ for all those households with a positive level of optimal saving.

Suppose $\phi > \hat{\phi}$. One can easily verify that, if and only if $\pi < \overline{\pi}$, there exists a non-empty interval of human capital level $[h, \overline{h}]$ such that $\lambda_i > 0$ if and only if $h_i \in (h, \overline{h})$, where $\overline{\pi} = \frac{2}{3} \left[ \phi \left( \frac{1-\beta}{\beta} \right) + \frac{1}{2R} - 1 \right]$, $\overline{h} \in (h_L, h_H)$ and $\overline{h} > h_H$. That is, if the credit market imperfections are sufficiently weak, i.e. $\pi \geq \overline{\pi}$, there are no borrowing constraints in the economy.

Whenever $\pi < \overline{\pi}$ and $\lambda_i > 0$, the optimal level of schooling at time $t$ is given by

$$e_i = \frac{1}{1+\delta} \left[ \frac{\delta \beta}{\phi} (1+\phi+\pi) - \frac{1}{\theta} \left( \frac{\beta}{(1-\beta) \phi pwh_i} \right)^{1-\beta} \right],$$

(52)

where $\partial e_i/\partial \pi > 0$. That is, for those households restricted by the borrowing constraint, the optimal choice of schooling decreases with the strength of the credit market imperfections.

Next, since $\pi$ affects the optimal allocation of time in school, it also affects the threshold level $h_H$ that determines whether a household has its child attending school full time. From equation (52) we have that $e_i = 1$ if and only if $h_i \geq h'_H$, with

$$h'_H = \frac{\beta}{(1-\beta) \phi pwh} \left[ \frac{\phi}{\theta [\delta \beta (1+\phi+\pi) - (1+\delta) \phi]} \right]^{1-\beta},$$

(53)

where $h'_H > h_H$ if and only if $\phi > \hat{\phi}$, and with $\partial h'_H/\partial \pi < 0$. That is, the threshold $h'_H$ increases with the strength of the credit market imperfections.

Figure 5 presents the optimal household’s saving for two different cases: $\pi > \overline{\pi}$ and $\pi = 0$. Notice from the figure that in the short run, borrowing constraints decrease the proportion of children attending school full time, increasing in turn, the proportion of children working part-time. That is, credit market imperfections tend to decrease the allocation of time in school in the short run whenever the productivity of child labor is sufficiently large. In the long run, however, since $\pi$ does not affect the threshold $h_L$ the economy converges to the same optimal allocations as before.
5.3 Inequality

In this Section we briefly discuss some implications of differences in inequality on the distribution of human capital. To maintain the analysis as simple as possible, the concept of inequality used here is based on the concept of generalized Lorenz dominance.\footnote{This concept is based on the generalized Lorenz curve introduced by Shorrocks (1983), where the ordinates of the generalized Lorenz curve are the standard Lorenz curve ordinates multiplied by the average level of the distribution.} The convenience of this approach relies on the fact that the concept of generalized Lorenz dominance is equivalent to the concept of second-order stochastic dominance. In particular, a distribution $A$ (generalized) Lorenz dominates another distribution $B$ if and only if the distribution $A$ dominates $B$ stochastically at a second order (i.e. inequality in $B$ is larger). Formally, distribution $A$ dominates $B$ stochastically at a second order if $\int_{-\infty}^{m} [B(x) - A(x)] \, dx \geq 0$ for all $m$, and $\int_{-\infty}^{m} [B(x) - A(x)] \, dx > 0$ for some $m$.

With the concept of second-order dominance in place, it is straightforward to examine the implications of inequality in the context of this model. Suppose we start at time 0 with two different distributions of human capital in the economy, $G_1(h)$ and $G_2(h)$, continuously defined over the same support and with the same initial mean, and where the distribution $G_2(h)$ strongly dominates the distribution $G_1(h)$ stochastically at a second order, i.e. in-
equality in $G_1(h)$ is greater. Notice that this assumption implies that the (standard) Lorenz curve for distribution $G_1(h)$ lies below the Lorenz curve for distribution $G_2(h)$.

We have then $G_1(h_L) > G_2(h_L)$ and $1 - G_1(h_H) < 1 - G_2(h_H)$. That is, the proportion of households with their children out from school is greater in the economy with larger inequality, while the proportion of households with children attending school full time is greater in the economy with lower inequality.

Since $G_1(h_L) > G_2(h_L)$, in the long run, the level of child crime is larger in the economy with greater inequality. Furthermore, given that $G_1(h_H) > G_2(h_H)$, notice from equations (37) and (38) that, even though both economies converge to the same growth rate in the limit as $t \to \infty$, the growth rate in the economy with larger inequality is lower during the transition.

5.3.1 Poverty and child labor in the cross-section

Cross-country studies such as Krueger (1997), and Dessy and Knowles (2008), find a strong negative correlation between child labor participation (10-14 years old) and GDP per capita. Many other cross-country studies trying to relate child labor and poverty find similar results. However, the role of poverty is not so predominant in explaining variations within a community.

Cross-section studies within a country tend to find a positive relationship between child labor and poverty in middle income countries, but no significant relationship in low income countries. For instance, Cartwright (1999) analyzing 1993 survey data for rural and urban children in Colombia finds that poverty plays a central role in driving children to work. Ray (2000) and Cartwright and Patrinos (1999) come to a similar conclusion using data from Peru and Bolivia in 1993, respectively. However, for the case of low income countries, Coulombe (1998) finds for Cote d’Ivore that income, once corrected for endogeneity, plays a small role in determining child labor. Canagarajah and Coulombe (1998) analyzing the work-school choices in Ghana in 1991-1992, find that household income does not play a significant role in child labor. In fact, they find that child labor falls with income only in the upper range of the income distribution. This pattern is also found in Bhalotra and Heady (1998) for the
In this Section, I use the above model to discuss a simple example that gives a rationale for this pattern. In the model, the lower the level of human capital of a household, the lower household’s earnings are. In turn, the lower household’s earnings are, the more likely the children will participate in activities different from school.

But how schooling and poverty are related at the country level in this economy? Following a simple measure of poverty such as the head-count ratio, poverty at any time $t$ in this economy can be defined by the density of the population below some threshold level of household’s human capital. Hence, ceteris paribus, country 1 is poorer relative to country 2 if the distribution of human capital in country 2 dominates the distribution of human capital in country 1 stochastically at a first order.

To exemplify the relationship between poverty and the activity of the children at some particular time $t$, suppose we have three countries differing only in their distribution of human capital. Country $j = 1, 2, 3$ is defined by its distribution of human capital with density function $g_j(h_{i,t})$, such that the distribution in country 3 $(G_3(h))$ dominates the distribution $G_2(h)$ stochastically at a first order, and the distribution $G_2(h)$ in turn dominates the distribution $G_1(h)$ stochastically at a first order (country 1 is relatively poorer than country 2, and this one is relatively poorer than country 3). The function $e_{i,t}(h_{i,t})$ and the density functions for the three countries are showed in Figure 6.

Figure 6: Poverty and the allocation of time

![Figure 6: Poverty and the allocation of time](image)

In Figure 6, the vertical axis presents the proportion of time a child devotes to school, and the horizontal axis presents the level of human capital of the household. Country 1,
represented by $g_1(h_{i,t})$ has a larger proportion of its population not allocating school-time to the children; country 2, represented by $g_2(h_{i,t})$, has a larger proportion of its population allocating partial school-time to the children; and country 3, represented by $g_3(h_{i,t})$, has a larger proportion of its population allocating full school-time to the children.

The results from the model suggest that if we consider a cross-section of countries, it is possible to find a negative relationship between poverty and school attendance. However, this hypothesis should be tested with caution if we consider a cross-section study of a particular economy. To see this, take for example the different countries in Figure 6. Hypothetically, if we run a regression with data for school attendance and households' income from Country 1 (assuming we are able to control for all possible covariates), what we would probably observe in the statistical result is that changes in income are not associated with changes in school attendance, since for the households with $h_{i,t} < h_L$ (a relatively large fraction of the population), the choice for school attendance is the same, $e_{i,t} = 0$. This result could mislead the interpretation by apparently showing that school attendance and poverty are not related, which in this case is not correct. The reason is that all those households with lower levels of human capital than $h_L$ are in a corner solution for the choice of school attendance. The same would be true for the case of Country 3. Actually, only the data from Country 2 would be likely to show a statistically significant relationship between schooling and poverty.

This simple example depicts why we may observe, as in some empirical cases, that there is no statistically significant relationship between poverty and school attendance in low-income countries, while there is a significant relationship for middle-income countries.

6 The closed economy case

As mentioned above, the small open economy assumption simplifies the analysis by eliminating dynamics in the determination of the physical capital-effective human capital ratio. In this section, I depart from this assumption and consider the case of a closed economy.

Nevertheless, to simplify the analysis and in order to obtain an analytical solution to the model, it is assumed that old individuals do not work. Thus, only adults face non-trivial
decisions having preferences of the form

\[ U = u(c_{ai}) + u(c_{oi}') + \delta v(h_i') \equiv \ln(c_{ai}) + \ln(c_{oi}') + \delta \ln(h_i') , \]  

(54)

where, \( c_{ai} \) is the household \( i \)'s current consumption, \( c_{oi}' \) is individual \( i \)'s consumption when old, and \( h_i' \) is the level of human capital of individual \( i \)'s offspring once she is an adult.

When old, individuals simply consume their capital income

\[ c_{oi}' = R's_i, \]  

(55)

where in this case, the rental rate of capital is not determined in the international capital market, but by the equilibrium condition given in equation (10). Otherwise, the economy model is as before.

From the production technology in equation (5), the aggregate stock of effective human capital, \( H \), is now given by

\[ H \equiv H_a + \phi H_c = \int_{0}^{1} h_i di + \phi \int_{0}^{1} x_i h_i di, \]  

(56)

where \( H_a \) is the aggregate stock of human capital provided by adults, \( H_c \) is the aggregate stock of human capital provided by children.

A competitive equilibrium is a sequence of allocations \( \{e_i(t), x_i(t), z_i(t), b_i(t), s_i(t)\}_{t=0}^{\infty} \) for all individuals \( i \in [0,1] \), a sequence of distributions of human capital \( \{G_t(h_i(t))\}_{t=0}^{\infty} \) with \( g_0(h_i(0)) \) given, a sequence of allocations \( \{K(t)\}_{t=0}^{\infty} \), with \( K(0) = K_0 > 0 \), and a sequence of prices \( \{R(t), w(t), w_c(t)\}_{t=0}^{\infty} \) such that, given prices, each individual maximizes her utility, firms maximize profits, human capital for each individual evolves according to (3), with \( h_i(0) = h_{i0} > 0 \) for all \( i \in [0,1] \), the distribution of crime rents satisfies (9), and every market clears for all \( t \geq 0 \).

In order to focus our attention in the more interesting case, I assume that the initial stock of physical capital satisfies \( K_0 < h_0 [(1 - \alpha)A/2]^{1/(1-\alpha)} \). This assumption ensures that the \( K/H \) ratio increases over time.

The equilibrium analysis of this Section parallels that of Section 3. In this case, however,
the market clearing condition in the physical capital market implies that every period aggregate savings and aggregate investment in physical capital are equal, i.e. \( K' = \int_0^1 s_i \, di \). In addition, the wage rate is not constant over time and it is not determined by equation (15). Instead, from equation (11), the wage rate for adult labor every period is given by

\[
w = (1 - \alpha) A \left( \frac{K}{H} \right)^\alpha,
\]  
(57)

with \( w_c = \phi w \). Similarly, the gross interest rate every period is given by

\[
R = \alpha A \left( \frac{H}{K} \right)^{1-\alpha}.
\]  
(58)

Following the same steps as before, the Euler equation together with \( c_i' = R's_i \) imply \( s_i = c_i \) for all \( i \) and for all \( t \). Then, optimal saving for all \( i \) is given by the general expression

\[
s_i = \frac{1}{2} npwh_i \left[ 1 + \phi (1 - e_i) + \frac{(1-p)Y_L}{npwh_i} - \frac{b_i}{pw_i} \right],
\]  
(59)

where \( s_i > 0 \) for all \( i \). In addition, one can easily verify that the optimal level of schooling at time \( t \) is given by

\[
e_i = \frac{2}{2 + \delta} \left[ \frac{\delta \beta}{2\phi} (1 + \phi) - \frac{1}{\theta} \left( \frac{\beta}{(1 - \beta) \phi pw_i} \right)^{1-\beta} \right],
\]  
(60)

where \( w \) is given by equation (57). Thus, we have that \( e_i = 0 \) if and only if \( h_i \leq \hat{h}_L \), with

\[
\hat{h}_L = \frac{\beta}{(1 - \beta) \phi pw} \left[ \frac{2\phi}{\theta \delta \beta (1 + \phi)} \right]^{1/\gamma}.
\]  
(61)

Similarly, \( e_i = 1 \) if and only if \( h_i \geq \hat{h}_H \), with

\[
\hat{h}_H = \frac{\beta}{(1 - \beta) \phi pw} \left[ \frac{2\phi}{\theta [\delta \beta (1 + \phi) - (2 + \delta) \phi]} \right]^{1/\gamma},
\]  
(62)

where the right-hand-side of this expression is defined if and only if \( \delta > \hat{\delta}_L \equiv \frac{2\phi}{\theta (1 + \phi) - \phi} \), with \( \hat{\delta}_L > 0 \) since \( \beta > 1/2 \). Thus, whenever \( \hat{h}_H \) exists, we have \( \hat{h}_H > \hat{h}_L \). Notice from equations (57), (61) and (62) that both \( \hat{h}_H \) and \( \hat{h}_L \) decrease over time if and only if the ratio \( K/H \)
increases over time.

From equation (22), the optimal investment on education for all \( h_i \in [\hat{h}_L, \hat{h}_H] \) continues to be given by equation (23), and for all \( h_i > \hat{h}_H \) it is now given by the unique solution to the following expression:

\[
\frac{2}{\theta} b_i^\beta + (2 + \delta (1 - \beta)) b_i = \delta (1 - \beta) p w h_i,
\]

(63)

with \( db_i/dh_i > 0 \) and \( db_i/d\theta > 0 \). As before, it is convenient to assume that \( b_i \) is bounded by some level \( b_H \).

Next, from the optimality of child crime we have that \( z_i = 1 \) if and only if \( h_i \geq \hat{h}_c \), where \( \hat{h}_c \) is given by the unique solution to the following expression

\[
\hat{h}_c G (\hat{h}_c) = \frac{(1 - \varepsilon)^{\delta/2} (1 - p) H}{p \left[ 1 + \phi - (1 - \varepsilon)^{\delta/2} \right]},
\]

(64)

where \( G(\hat{h}_c) \in (0, 1) \) is the proportion of child crime in the economy. As before, one can verify that \( \hat{h}_c < h_L \) if and only if \( p > \hat{p}_L \), with \( \hat{p}_L < 1 \). In addition, the threshold \( \hat{h}_c \) increases over time whenever the aggregate stock of effective human capital, \( H \), increases over time.

Finally, from equation (59), aggregate investment every period is determined by

\[
K' = \frac{1}{2} \left[ (1 - \alpha) AK^\alpha H^{1-\alpha} - \int_0^1 b_i d\bar{z} \right].
\]

(65)

In the long run, the economy converges asymptotically to a unique steady state equilibrium where the ratios \( Y/H, K/H, \) and the growth rate of the economy are all constant over time. In particular, one can verify that \( \lim_{t \to \infty} H_t = \int_0^1 h_{i,t} d\bar{z} = \hat{h}_t \), and then we have

\[
\lim_{t \to \infty} \frac{K_t}{H_t} = \left[ \frac{(1 - \alpha) A}{2 (1 + \theta b_H^{1-\beta})} \right]^{1/(1-\alpha)},
\]

(66)

\[
\lim_{t \to \infty} \frac{Y_t}{H_t} = A^{1/(1-\alpha)} \left[ \frac{1 - \alpha}{2 (1 + \theta b_H^{1-\beta})} \right]^{\alpha/(1-\alpha)},
\]

(67)
\[
\lim_{t \to \infty} \frac{\Delta Y_t}{Y_t} = \lim_{t \to \infty} \frac{\Delta K_t}{K_t} = \lim_{t \to \infty} \frac{\Delta H_t}{H_t} = \theta b_H^{1-\beta}.
\] (68)

Figure 7 illustrates the transitional dynamics of the allocations of the children’s time in the closed economy case. As in Figure 3, the vertical axis denotes the mass of children conveniently indexed and the horizontal axis denotes time.

Figure 7: Allocation of children’s time in a closed economy

Different from the open economy case, in the closed economy case there exists a non-empty set of dynasties in which the children start by working full time, and in the long run, the corresponding children of the same dynasty attend school full time (see example (a) in Figure 7). In this case, as the economy develops, the $K/H$ ratio in the economy increases over time increasing in turn the wage rate. The progressive rise in the household’s adult labor earnings together with the increase on the child labor wage rate, allow the child of the corresponding dynasty to attend school part time at some point in the future. Following this process, the proportion of the child’s time devoted to school increases in the following generations, up to the point where the child will attend school full time.

Using the above model of a closed economy, we have the following result.

**Proposition 4** If $p < \hat{p}_H$ and $\theta < \hat{\theta}_L$, a temporary ban on child labor has permanent negative effects in the long run for all individuals in the economy.

That is, different from the open economy (see Proposition 2), in this case a temporary ban can have permanent negative effects not only for the children living in the poorest
households, but for all individuals in the economy.

To see this, suppose a temporary ban is implemented at some time \( t \) and it is abrogated at \( t + 1 \). Given that the analysis from this Proposition parallels that of Proposition 2, I shall discuss only the new results.

Consider first the contemporaneous effects. At the time of the implementation, the aggregate stock of effective human capital \( H \) falls with the ban since there are no working children. This effect increases the \( K/H \) ratio since the existing stock of physical capital \( K \) is already determined, which in turn increases the wage rate for adult labor \( w \) and the household’s adult labor earnings. Therefore, contemporaneously, those households with the largest levels of human capital, \( h_i \geq \hat{h}_H \) (i.e. with children attending school full time), are unambiguously better off with the ban because the increase on adult labor earnings increase current consumption.

Next, even though the wage rate for adult labor increases, aggregate labor income, \( Y_L = (1 - \alpha) Y \), falls with the ban. In turn, the decrease in \( Y_L \) induces a fall on aggregate investment relative to the no-ban case. Then, the aggregate stock of physical capital at \( t + 1 \) is relatively lower due to the ban.

Now, once the ban is abrogated at time \( t + 1 \), the aggregate stock of effective human capital \( H \) is lower relative to the no-ban case. This comes from the fact that, if the level of effective property rights and the quality of education are sufficiently low, the temporary ban increases permanently the level of child crime in the economy. Furthermore, \( H \) is lower relative to the no-ban case every period. This, in turn, generates also physical capital and output to be lower relative to the no-ban case.

Relative to the no-ban case, if \( p < \hat{p}_H \) and \( \theta < \hat{\theta}_L \) the fall in the aggregate stock of effective human capital induces a more than proportional decrease on aggregate investment in the future. Together, these two effects induce a fall on the \( K'/H' \) ratio. Then, even though the economy after the temporary ban on child labor converges in the limit as \( t \to \infty \) to the same ratios \( K/H \) and \( Y/H \) given in equations (66) and (67), respectively, and the same growth rate given in equation (38), it does so from below. That is, every period these ratios and the growth rate of the economy are lower due to the temporary ban.
Finally, the decrease on the $K/H$ ratio implies a reduction on the wage rate for adult labor $w$. Therefore, in the long run not only the level of child crime is greater because of the temporary ban, but in addition, consumption decreases for all households in the economy because of the reduction on adult labor earnings.

7 Conclusion

This paper analyzes the role that institutional quality — security of property rights, quality of education, and credit market imperfections — plays on the relationship between child labor, child crime, and schooling, in an economy with income inequality. The study illustrates the important role of the institutional quality and the distribution of human capital in the activity of children. It shows that the interplay between economic institutions and the incentives households face when allocating their children’s time greatly shapes the consequences of a child labor ban.

The study suggests that, in economies where the quality of education and the security of property rights are relatively low, a ban on child labor tends to harm the very same children it is intending to help: the working children living in the poorest households. The reason is that a policy designed to ban child labor exacerbates the problem of child crime by leading these working children into criminal activities. Furthermore, even a temporary ban on child labor can have permanent negative effects for all individuals in the economy if the security of property rights and the quality of education are sufficiently low.

A further implication of the model is that child labor is not eliminated from the economy by having perfect capital markets, although it might be reduced. On the other hand, a policy that increases the quality of education may completely eliminate child labor, even in the presence of credit market imperfections. The results also suggest that an economy may find it beneficial to invest on institutional quality if this investment is financed by households with the largest levels of income, or if the efficacy of the investment to increase the quality of institutions in the economy is sufficiently large.
Appendix

Proof of Proposition 1

Consider Part (i). One can verify that, every time $t$, utility for individual $i$ is equal to $\ln \left[ B_1 c^3_{ai} (h'_i)^d / h_i \right]$, where $B_1 > 0$ is some constant. Then, a household will prefer child crime over full-time child labor if and only if $\left. \frac{c^3_{ai}(h'_i)^d}{h_i} \right|_{z_i=1} \geq \left. \frac{c^3_{ai}(h'_i)^d}{h_i} \right|_{x_i=1}$. Evaluating equations (3) and (20) at $z_i = 1$ and $x_i = 1$, respectively, it is easy to verify that $z_i = 1$ if and only if $h_i \leq h_c$ where $h_c$ is the solution to equation (29). Since $h_c G(h_c)$ is strictly increasing in $h_c$ for all $h_c \geq 0$, with $h_c G(h_c) = 0$ when $h_c = 0$ and $h_c G(h_c) \rightarrow 0$ when $h_c \rightarrow 0$, there exists a unique solution to equation (29).

Consider Part (ii). From the analysis leading to Proposition 1, we have that $x_i = 1$ if and only if $h_i < h_L$ where $h_L$ is given by equation (25). Next, one must ensure that $h_c < h_L$. From equations (25) and (29) one can verify that $h_c \leq h_L$ if and only if $p \geq 1 - \Psi_1 \equiv p_L$, where $\Psi_1$ is some positive number.

Parts (iii) and (iv) follow from the analysis leading to Proposition 1. This concludes the proof. QED

Proof of Proposition 2

Consider Part (i). After the imposition of the full ban on child labor, as noted in the proof of Proposition 1, a household will prefer child crime over full-time schooling if and only if $\left. \frac{c^3_{ai}(h'_i)^d}{h_i} \right|_{z_i=1} \geq \left. \frac{c^3_{ai}(h'_i)^d}{h_i} \right|_{e_i=1}$. Evaluating equations (3) and (20) at $z_i = 1$ and $e_i = 1$, one can verify that $z_i = 1$ if and only if $h_i \leq h'_c$ where $h'_c$ is the unique solution to the following expression

$$h'_c \quad G(h'_c) = \frac{(1 - \varepsilon)^{\delta/3} (1 - p) H \left( 1 + \frac{b_H G(h'_c)}{1-p} w_H \left( \frac{1+\theta b^1_H - \beta}{1-\varepsilon} \right)^{\delta/3} \right) \left( 1 + \theta b^1_H \right)^{\delta/3} - (1 - \varepsilon)^{\delta/3}}{p \left( 1 + \frac{1}{R} \right) \left( 1 + \theta b^1_H \right)^{\delta/3} - (1 - \varepsilon)^{\delta/3}}, \quad (69)$$

where one can easily verify that $h'_c \in (h_c, h_H)$. Next, a permanent ban increases the proportion of child crime in the long run only if $h'_c > h_L$. Comparing equations (29) and (69), one can verify that $h'_c > h_L$ if and only if $p < p_H \equiv 1 - \Psi_2 + \varphi > 0$, with $p_H > p_L$ because $\Psi_2 < \Psi_1$ whenever $\varepsilon \in \left[ 0, \bar{\varepsilon} \right]$, and $\varphi$ is some positive number, with $\bar{\varepsilon} \equiv 1 - \frac{\delta \beta (1+\phi+1/R) - \delta}{(2+\phi/(1+1/R))^{\delta-\delta} [\delta \beta (1+\phi+1/R) - (3+\delta) \phi]} \in (0,1)$. Noticing that the function $h'_i$ is con-
continuous and increasing in \( \theta \) for all \( \theta > 0 \), with \( h'_i/h_i = 1 \) when \( \theta = 0 \) and \( h'_i/h_i \to \infty \) as \( \theta \to \infty \), the rest of Part (i) follows from Panel (a) of Figure 4.

Parts (ii) to (iv) follow from the fact that \( h'_c > h_L \) if and only if \( p < p_H \), and from the analysis of the Panel (b) of Figure 4 in the text. This concludes the proof. QED

**Proof of Proposition 3**

Consider Part (i). From equation (26), the proportion of children attending school full time increases since \( \partial h_H/\partial \theta < 0 \). From equations (25) and (29), the proportion of children working full time decreases since \( \partial h_L/\partial \theta < 0 \) and \( dh_c/d\theta = 0 \). In the long run, both the time it takes to eliminate child labor from the economy and the level of child crime decreases since \( \partial h_L/\partial \theta < 0 \).

Consider Part (ii). The proportion of children attending school full time increases since \( \partial h_H/\partial p < 0 \). The proportion of children out from school decreases since \( \partial h_L/\partial p < 0 \). The level of child crime in the short run decreases since \( dh_c/dp < 0 \), and also in the long run since \( \partial h_L/\partial p < 0 \). This concludes the proof. QED

**Proof of Proposition 4**

At the time of the temporary ban, one can verify that \( z_i = 1 \) if and only if \( h_i \leq \tilde{h}'_c \) where \( \tilde{h}'_c \) is given by the unique solution to the following expression

\[
\tilde{h}'_c G(\tilde{h}'_c) = \frac{(1 - \varepsilon)^{\delta/2} (1 - p) H \left( 1 + \frac{b_H G(\tilde{h}'_c)}{(1-p)wH} \left( \frac{1 + \theta \theta'_L}{1 - \varepsilon} \right)^{\delta/2} \right)}{p \left[ (1 + \theta \theta'_L)^{\delta/2} - (1 - \varepsilon)^{\delta/2} \right]},
\]

with \( \tilde{h}'_c \in (\tilde{h}_c, \tilde{h}_H) \). From equations (61) and (70) one can verify that there exists a level of effective property rights \( \hat{p}_H > 0 \) such that \( \hat{h}_L > \tilde{h}_L \) if and only if \( p < \hat{p}_H \), with \( \hat{p}_H > \hat{p}_L \).

Now, following the same arguments presented in the proof of Proposition 2, there exists a level \( \hat{\theta}_L \) such that a temporary ban increases the level of child crime in the long run if and only if \( p < \hat{p}_H \) and \( \theta < \hat{\theta}_L \). This concludes the first part of the proof.

Next, we prove that \( \tilde{K}'_H > \tilde{K}'_H \), where \( \tilde{K}'_H \) is the physical capital-human capital ratio next period if no ban were implemented. Recalling that \( h = \int_0^1 h_i d_i \), from equation (65) one can
verify that \( \frac{K'}{H'} > \frac{\tilde{K}'}{\tilde{H}'} \) if and only if

\[
\frac{(1 - \alpha) AK^\alpha H^{1-\alpha} - \int_{b_i}^\infty b_i di}{(1 - \alpha) AK^\alpha h^{1-\alpha} - \int_{b_i}^\infty b_i di} > \frac{h' + \phi \left[ G(h_L) - G(h'_L) \right] + \phi \int_{h_L}^{H'} x'_i di}{\tilde{h}' + \phi \left[ G(h_L) - G(\tilde{h}_L) \right] + \phi \int_{h_L}^{H'} \tilde{x}'_i di},
\]

(71)

where \( h' = h + \theta \left[ \int_{h_L}^{H'} \epsilon_i b_i 1^{-\beta} h_i di + \int_{h_L}^\infty b_i 1^{-\beta} h_i di \right] - \varepsilon G(h_L) \) and \( \tilde{h}' = h + \theta \int_{h_L}^{\infty} 1^{-\beta} h_i di - \varepsilon G(\tilde{h}_L) \) with \( \tilde{h}' > h' \). By substituting these two last expressions into equation (71) one can verify that the inequality always hold whenever \( \varepsilon < \tilde{\varepsilon}, p < \tilde{p}_H, \) and \( \theta < \tilde{\theta}_L, \) where \( \tilde{\varepsilon} \) is the constant given in the proof of Proposition 2. Finally, note that if \( \frac{K'}{H'} > \frac{\tilde{K}'}{\tilde{H}'} \) at some time \( t \), then \( \frac{K'}{H'} > \frac{\tilde{K}'}{\tilde{H}'} \) for all \( t + a \) for any \( a \geq 1 \), with \( \lim_{t \to \infty} \frac{K_t}{H_t} = \lim_{t \to \infty} \frac{\tilde{K}_t}{\tilde{H}_t} \). That is, the difference decreases over time. This concludes the proof. QED
References


Gonzalez, F.M. and I. Rosales (2013): “Naive policies against child labor can harm the children of developing nations”, *Unpublished manuscript*.


