Staggered prices, the optimizing taylor rule and the irrelevance of the is curve

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ABSTRACT

When the central bank minimizes a loss function depending upon the variability of inflation and the variability of output, the resultant interest rate policy rule is the so-called Taylor rule. In this context, the form and the parameters of the IS curve- whether this one is the old Keynesian function or a version of the Euler equation in output (the new IS)- are irrelevant in the determination of inflation and output. The solution of the model shows that the expected value of output is the natural one and the expected value of inflation is the target of the central bank. There is a tradeoff between the variances of these variables (inflation and output), nonetheless. Output and the real rate of interest will be more variable the more the central bank worries about the stability of inflation around its target.

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STAGGERED PRICES, THE OPTIMIZING TAYLOR RULE AND THE IRRELEVANCE OF THE IS CURVE.

INTRODUCTION

Staggered prices became an important topic since the original research by Taylor (1979) (1980) and then by Calvo (1983). From that moment onwards the idea has evolved albeit some critiques. One of the main results of the approach is the emergence of the so-called new Phillips curve: a function where basically present inflation is related positively to future expected inflation and the actual output gap.

On the other hand, the premise that all agents in the economy act rationally has been gaining adepts at very high rates, which happens despite different theoretical and empirical works showing that rationality is not always a fact.

The combination of the new Phillips curve, an optimizing central bank, and also rational forward looking private agents, generate a situation where there is a tradeoff between the variance of inflation and the variance of output. Output will be more variable- and inflation less- the more the central bank worries about the stability of inflation around a definite target.

In the construction of the model with characteristics already explained, a striking result is the irrelevance of the form and the parameters of the IS curve in the determination of inflation and output. These two variables will have the same value and the same probability distribution independently of the IS curve.

People may be rational and the IS curve then will be a transformation of the Euler equation of consumption, or there could be some irrationality-or liquidity constraints- where people consume some fixed proportion of their disposable income. Under certain conditions that

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2 See Akerlof, Dickens and Perry (2000). Also it is not clear whether or not staggered prices may constitute in itself a rational decision (see for example Deveraux and Yeatman (2002)).
does not matter, the solution for inflation and output will be independent of the nature of the IS function.

Another possible new result in this work is the one of the variance of the real rate of interest. The more the central bank cares about inflation approaching to its target, the higher is the variance in the real interest rate. The size of the variance of the real interest rate does not depend upon of the nature of the IS function (whether there is an old Keynesian IS curve or a new Euler equation IS) but of the response of output to the real interest rate in that function.

This paper sets a very simple model of staggered prices where the central bank follows an optimizing Taylor rule for interest rates. It tries to explain why in this context the IS curve is irrelevant to determine inflation and output. It also reflects on the results and in the social validity of the loss function set by the central bank.

I. THE MODEL

We set a very simple model where prices are staggered, generating the simplest possible new Phillips curve, consumers may or may not act in a fully rational way and the central bank, or the institution responsible for the macroeconomic stability of the economy, act always rationally.

The simplest new Phillips curve is the one derived by Mankiw and Reis (2001) based in the work by Calvo (1983), which may be described as:

\[ \pi_t = E_t \pi_{t+1} + \delta (y_t - y^*) + e_t \]  
(1)

Where \( \pi_t \) is the actual rate of inflation; \( E_t \pi_{t+1} \) is the conditional expected value of inflation in the next period. The variable \( y_t \) is present output and \( y^* \) is potential output. Therefore \( y_t - y^* \) is the output gap. The term \( e_t \) is an independent normally distributed supply random shock with zero mean and constant variance \( \sigma^2 \).

There is a IS curve in the economy, which we define as:

\[ y_t = H - br_t + v_t \]  
(2)
This function establishes a consistent negative relation between income \((y)\) and the real rate of interest \(r\). Parameter \(v\) is an independent random shock normally distributed with zero mean and variance equal to \(\sigma_v^2\).

We analyze two types of IS curves:

The first assumes Keynesian characteristics: The parameter \(H\) in (2) is related to fiscal parameters, like government expenditure and the rate of income tax. It is also related to private parameters: The autonomous private expenditure, for instance. In this way \(H\) becomes:

\[
H = f(G, \tau, C_0) \quad (3)
\]

Parameter \(G\) is government expenditure, \(\tau\) is the income tax rate and \(C_0\) is private autonomous expenditure. In this case \(df/dg>0; \, df/d\tau<0\) and \(df/dC_0>0\)

The IS curve may be then described as:

\[
y_t = f(G_t, \tau, C_0) - br_t + v_t \quad (4)
\]

A second form of the IS curve is the one that emerges from the Euler equation in consumption. Under certain circumstances there is also an Euler equation for output (see for instance McCallum and Nelson (1999), King (2000), Woodford (2001) and Blanchard (2008)). The new IS curve may be estimated (See Rotemberg and Woodford (1998) or Fuhrer and Rudebusch (2004)).

In the case of the new IS curve, \(H\) becomes:

\[
H = E_t y_{t+1} + b\theta \quad (5)
\]

Where \(E_t y_{t+1}\) is the conditional expected value on present time of output in the next period. \(\theta\) is the subjective discount factor of the utility function for consumers.

Substituting (5) in the generic IS curve (2), we get the new IS curve:

\[
y_t = E_t y_{t+1} - b(r_t - \theta) + v_t \quad (6)
\]
The central bank, or the institution in charge of macroeconomic stability, sets an inflation target ($\pi^*$) and also pretends to maintain output as near as possible to the normal or natural level $y^*$, which is the level where inflation remain constant according to the new Phillips curve (1).

The loss function that the central bank is aimed to minimize is:

$$L_t = \varphi(\pi_t - \pi^*)^2 + (1 - \varphi)(y_t - y^*)^2 \quad (7)$$

The function is quadratic as in the Barro-Gordon approach (Barro and Gordon (1983)).

The best possible outcome for the central bank is to have actual inflation $\pi_t$ equal to the inflation target $\pi^*$ and present output $y_t$ equal to natural output $y^*$. Every other possibility produces a positive loss function.

The parameter $\varphi$ is a representation of how important is for the central bank the stability of inflation around its target in comparison to the stability of output ($0 < \varphi < 1$). When $\varphi$ is 1 the central bank is just committed to set inflation in its target. On the contrary, when $\varphi=0$ ($1-\varphi=1$) the central bank just cares about the stability of output. In practice the central bank cares about the two objectives though in some countries the law indicates that the institution must care just about to set inflation in its target.

However, the central bank has a restriction to minimize (7), the Phillips curve (1).

II.- MINIMIZATION OF THE LOSS FUNCTION: THE IRRELEVANCE OF THE IS CURVE.

The primal problem to solve is to minimize equation (7) of the loss function subject to equation (1) of the Phillips curve. That will generate a relation between the output gap ($y_t - y^*$) and the inflation gap ($\pi_t - \pi^*$). To get this function the central bank has to set a monetary rule for the rate of interest, which is the very well-known Taylor rule (see Taylor (1993)).

Minimization of (7) subject to (1) gives, as a result:

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3 An important difference of this approach with the Barro-Gordon version is that in the last one the target for income is greater than the natural output $y^*$.

4 Chile, Mexico, New Zealand
\[ y_t - y^* = \frac{\varphi \delta (E_t \pi_{t+1} - \pi^*)}{(\varphi \delta^2 + (1-\varphi))} - \frac{\varphi \delta e_t}{(\varphi \delta^2 + (1-\varphi))} \quad (8) \]

This equation, which we call the aggregate demand, shows a negative association between the output gap \((y_t - y^*)\) and a proxy of the inflation gap \((E_t \pi_{t+1} - \pi^*)\).

For equation (8) being in fact the aggregate demand, the central bank has to set a rule for interest rates. Substituting (8) in the generic IS curve (2) we get:

\[ r_t = \frac{H - y^*}{b} + \frac{\varphi \delta (E_t \pi_{t+1} - \pi^*)}{(\varphi \delta^2 + (1-\varphi))b} + \frac{\varphi \delta e_t}{(\varphi \delta^2 + (1-\varphi))b} + \frac{v_t}{b} \quad (9) \]

Equation (9) is the generic Taylor rule in real interest rates (the MP equation of the Romer-Taylor model. See Romer (2000), Taylor (2000)).

When the IS curve is of the Keynesian type, the Taylor rule becomes:

\[ r_t = \frac{f(G_t, \tau, \xi_0) - y^*}{b} + \frac{\varphi \delta (E_t \pi_{t+1} - \pi^*)}{(\varphi \delta^2 + (1-\varphi))b} + \frac{\varphi \delta e_t}{(\varphi \delta^2 + (1-\varphi))b} + \frac{v_t}{b} \quad (10) \]

In this case the monetary policy rule makes the real interest rate depend upon the inflation gap \((E_t \pi_{t+1} - \pi^*)\). An expansive fiscal policy generates a reaction where the central bank increases the nominal interest rate given the expectations of inflation for the next period. Positive supply shocks (e.g. \(\epsilon t < 0\)) reduce the real interest rate. Positive demand shocks (\(v_t > 0\)) generate an immediate increase of the interest rate.

When the relevant IS curve is the Euler equation for output (the new IS curve), the Taylor rule may be described as:

\[ r_t = \theta + \frac{E_t y_{t+1} - y^*}{b} + \frac{\varphi \delta (E_t \pi_{t+1} - \pi^*)}{(\varphi \delta^2 + (1-\varphi))b} + \frac{\varphi \delta e_t}{(\varphi \delta^2 + (1-\varphi))b} + \frac{v_t}{b} \quad (11) \]

The reaction function of the central bank makes the real interest rate to depend upon positively in both the output gap \((E_t y_{t+1} - y^*)\) and the inflation gap \((E_t \pi_{t+1} - \pi^*)\). Fiscal policy is irrelevant because there is Ricardian equivalence (see Barro (1974)). In equilibrium, when both the output gap and the inflation gap are zero, and when random supply and demand shocks are also zero:

\[^5\text{In this case, the Taylor rule for real interest rates does not depend upon the output gap.}\]
\[ r_t = \theta = r^* \quad (12) \]

The real rate of interest is equal to the subjective discount rate of the utility, which instead is the natural rate of interest, a concept introduced more than one hundred years ago by the Swedish economist Knut Wicksell (Wicksell (1898)).

The dual problem consist of taking the generic Taylor rule (9) and substitute it in the IS curve (2). The result is the aggregate demand (8). Clearly, the Phillips curve (1) and the aggregate demand (8) are the relevant equations to solve for inflation \( \pi_t \) and output \( y_t \), while the IS curve solves for the real interest rate \( r_t \).

It is irrelevant whether the IS curve has a Keynesian form, like the one of (4), or a form of the Euler equation like in equation (6). In both cases the resultant aggregate demand will be as in (8). In that aggregate demand fiscal policy does not matter in the solution of inflation and output. If the relevant IS function is of the Keynesian form, then fiscal policy certainly affects the long run value of the real rate of interest \( r_t \) but it does not affect the trajectories and probability distributions of inflation and output. Instead, when the relevant IS curve is the Euler equation (6), fiscal policy does not matter for the solution of any macroeconomic variable (output, inflation, the real rate of interest).

The irrelevance of the IS curve must be understood in the context of the solution of inflation and output. The IS curve is certainly relevant in the solution of the real interest rate. However, when the central bank minimizes the loss function (2), it sets the real interest rate in such a way that any shock coming from the IS is automatically cancelled.

The intuition behind this result is that the central bank tries to maintain the rate of inflation and output in the nearest possible vicinity of their long run targets \( (\pi^* \text{ and } y^*) \). A positive shock in the IS (for instance an increase in the parameter \( v \)) would increase aggregate demand, generating higher inflation and output in the short run. Then an increase in the rate of interest will offset the rise in the shock \( v \) (see equations (9), (10) and (11)).

When the central bank acts in an optimizing way, the solution for inflation and output does not depend in any way of the parameters of the IS curve. This result contrasts sharply with the examples provided by Romer (2000) and Taylor (2000) in the most simplified versions.
of the Romer-Taylor model. In those versions, changes in fiscal policy (Romer (2000)) modify output temporarily and inflation permanently. Also, changes in exports or imports, who influence the model through the IS curve, may have a permanent influence in output and inflation (Taylor (2000)).


The two equations necessary to solve for output and inflation are the new Phillips curve (1) and the aggregate demand (8), which we repeat below for convenience

\[ \pi_t = E_t \pi_{t+1} + \delta(y_t - y^*) + e_t \quad (13) \]

\[ y_t - y^* = \frac{\varphi \delta (E_t \pi_{t+1} - \pi^*)}{(\varphi \delta^2 + (1-\varphi))} \quad (14) \]

Substituting the output gap in (14) in the Phillips curve (13), we get a reduced form for inflation in the short run:

\[ \pi_t = \frac{(1-\varphi)}{\varphi \delta^2 + (1-\varphi)} E_t \pi_{t+1} + \frac{\varphi \delta^2}{\varphi \delta^2 + (1-\varphi)} \pi^* + \frac{(1-\varphi)}{\varphi \delta^2 + (1-\varphi)} e_t \quad (15) \]

Inflation depends upon its future expectations, the inflation target and the supply shock \( e_t \). In absence of random shocks inflation would be a weighted average of its future expectation and its target. None parameter of the IS curve is present in the short run reduced form for inflation.

To solve analytically the difference equation (16) we use the forward operator (see Hamilton (1994)).

\[ E_t \pi_{t+1} = L^{-x} \pi_t \quad (16) \]

Also, to simplify notation, we call

\[ j = \frac{\varphi \delta^2}{(\varphi \delta^2 + (1-\varphi))} \quad (17) \]

\[ 1 - j = \frac{(1-\varphi)}{(\varphi \delta^2 + (1-\varphi))} \quad (18) \]
Solution for inflation is then
\[ \pi_t = \pi^* + (1 - j) \sum_{i=0}^{\infty} (1 - j)^i E_t e_{t+i} \quad (19) \]

But
\[ \sum_{i=0}^{\infty} (1 - j)^i E_t e_{t+i} = e_t \quad (20) \]

Since \( E_t e_{t+i} = 0 \) for \( i > 0 \)

Then the definite reduced form for inflation is
\[ \pi_t = \pi^* + (1 - j)e_t \quad (21) \]

At the same time, to solve for \( y \) it is possible to set aggregate demand (8) as:
\[ y_t - y^* = -\frac{j}{\delta} (E_t \pi_{t+1} - \pi^*) + \frac{j}{\delta} e_t \quad (22) \]

But using (21) it is possible to show that:
\[ E_t \pi_{t+1} = \pi^* \quad (23) \]

Therefore, the reduced form for output is:
\[ y_t = y^* - \frac{j}{\delta} e_t \quad (24) \]

Positive supply shocks \( (e_t < 0) \) generate a situation where output rises above the natural level and inflation falls. However, given the assumptions of the model the impact of these shocks dilutes in just one period.

It is possible to show that there is a tradeoff between the variance of inflation and the variance of output. We will also show that output will be more variant the more worried is the central bank maintaining inflation around the target \( \pi^* \) and vice versa.

The non-conditional expectation for inflation in equation (21) is \( \pi^* \), since the non-conditional expectation for the disturbance term \( e_t \) is zero. Therefore, the variance of inflation is:
\[ Var(\pi_t) = E((\pi_t - E(\pi_t))^2) = E((\pi_t - \pi^*)^2) = (1 - j)^2 \sigma^2 \quad (25) \]
At the same time, the non-conditional expectation for output in (24) is $y^*$ because of the same reason that in the previous equation. Then

$$Var(y_t) = E(y_t - E(y_t))^2 = E(y_t - y^*)^2 = \frac{j^2}{\delta^2} \sigma^2 \quad (26)$$

Parameter $j$ is directly related to $\varphi$, which instead represents how much the central bank cares about the variance of inflation compared with the variance of output. When $\varphi=1$, then $j=1$ and the central bank cares just about maintaining inflation in its target $\pi^*$. When $\varphi=0$, $j=0$ and the central bank cares just about to maintain output at its natural level $y^*$.

There is a monotonic relation between $j$ and $\varphi$, which can be seen taking the derivative of $j$ with respect to $\varphi$ in (17):

$$\frac{dj}{d\varphi} = \frac{\delta^2}{(\varphi \delta^2 + (1-\varphi))^2} > 0 \quad (27)$$

$$\frac{dVar(\pi_t)}{dj} = -2(1-j)\sigma^2 < 0 \quad (28)$$

$$\frac{dVar(y_t)}{dj} = 2\frac{j}{\delta^2} \sigma^2 > 0 \quad (29)$$

The more the central bank cares to maintain inflation in its target $\pi^*$ (the higher are $\varphi$ and $j$), the lower is the variance of inflation and the higher is the variance of output. There is a tradeoff between the variance of inflation and the variance of output. The tradeoff between the variance of inflation and the variance of output is not new. In the paper by Erceg, Henderson and Levin (2000) this tradeoff implies the same that in this work: that the more the central bank cares about maintaining inflation in its target the higher is the variance of output.

The highest possible variance of output and the lowest possible variance for inflation occur when $\varphi=1=j$, in which case:

$$Var(\pi_t) = 0 \quad (30)$$

$$Var(y_t) = \frac{\sigma^2}{\delta^2} \quad (31)$$
The highest possible variance of inflation and the lowest possible variance for output occur when \( \varphi = 0 = j \), when

\[
Var(\pi_t) = \sigma^2 \quad (32)
\]

\[
Var(y_t) = 0 \quad (33)
\]

The variance’s function is a relation between the variance of inflation and the variance of output, which is found solving for \( j \) in (26) and substituting in (25), which gives, as a result:

\[
Var(\pi_t) = \sigma^2 - 2\left(Var(y_t)\right)^{1/2}\delta \sigma + Var(y_t)\delta^2 \quad (33)
\]

It is possible to prove that this function has a negative slope, since the first derivative of the variance of inflation with respect to the variance of output is:

\[
\frac{d(Var(\pi_t))}{d(Var(y_t))} = -\left(Var(y_t)\right)^{1/2}\delta \sigma + \delta^2 \quad (34)
\]

For this derivative being negative, it is necessary to have:

\[
-\left(Var(y_t)\right)^{1/2}\delta \sigma + \delta^2 < 0 \quad (35)
\]

But that means:

\[
\frac{\sigma^2}{\delta^2} > Var(y_t) \quad (36)
\]

This is true because the maximum variance of \( y \) is exactly \( \sigma^2/\delta^2 \).

At the same time, the second derivative is positive, showing that there is a convex relation in the plane where the variance of inflation is in the vertical axis and the variance of output is in the horizontal axis.

This can be seen taking the second derivative in (34).

\[
\frac{d^2 Var(\pi_t)}{d(Var(y_t))} = \frac{1}{2} \left(Var(y_t)\right)^{-3/2}\delta \sigma > 0 \quad (37)
\]
Which implies that in the plane where the variance of inflation is in the vertical axis, and the variance of output is in the horizontal axis, the relation between these two variables is convex to the origin.

On the other hand, the expected value of the loss function (7) is a linear relation between the variance of inflation and the variance of output, which may be seen applying the non-conditional expectation’s operator to equation (7) and considering that the non-conditional expectation for inflation and output are $\pi^*$ and $y^*$, respectively.

$$E(L_t) = \varphi E(\pi_t - \pi^*)^2 + (1 - \varphi)E(y_t - y^*)^2 = \varphi E((\pi_t - E(\pi_t))^2 + (1 - \varphi)E(y_t - E(y_t))^2 = \varphi Var(\pi_t) + (1 - \varphi)Var(y_t) \quad (38)$$

The expected result for the variance of inflation and the variance of output occurs in the tangency between the variance’s function (33) and the expected loss function (38). While there is a unique variance’s function, there is a dense map of expected loss functions, all of them linear (see figure 1).
Given the possible Taylor rules: the generic one in equation (9); the one appearing under the presence of the old Keynesian IS curve (equation (10)); and the one that surges under the Euler equation for output (11), the equilibrium solution for the real interest rate is:

\[ r_t = \frac{H-y}{b} + \frac{\varphi \delta e_t}{(\varphi \delta^2 + (1-\varphi))b} + \frac{v_t}{b} \quad (39) \text{ For the generic IS curve} \]

\[ r_t = \frac{f(G_t, T, G_0)-y}{b} + \frac{\varphi \delta e_t}{(\varphi \delta^2 + (1-\varphi))b} + \frac{v_t}{b} \quad (40) \text{ For the old Keynesian IS curve} \]

\[ r_t = \theta + \frac{\varphi \delta e_t}{(\varphi \delta^2 + (1-\varphi))b} + \frac{v_t}{b} \quad (41) \text{ For the new IS curve (Euler equation in output)} \]

In all of these cases, the non-conditional expectation for the real interest rate is:

\[ E(r_t) = r^* = \frac{H-y^*}{b} \quad (42) \text{ The natural rate of interest under the generic IS curve} \]
\( E(r_t) = r^* = \frac{f(G_t, \gamma, \delta)}{b} \) (43) The natural rate of interest under the old Keynesian IS curve

\( E(r_t) = r^* = \theta \) (44) The natural rate of interest under the new IS curve.

Then the variance of the real interest rate in all cases is:

\[
E(r_t - E(r_t))^2 = \text{Var}(r_t) = \frac{1}{b^2} \left( \frac{1}{\kappa^2} \sigma^2 + \sigma_v^2 \right)
\] (45)

The variance of the real interest rate depend upon both: the variance of the supply shocks \( e_t \) (\( \sigma_e^2 \)) and the variance of the IS shocks \( v_t \) (\( \sigma_v^2 \)). The higher the central bank cares about inflation, the higher is \( j \) and the higher is also the variance of the rate of interest. When the central bank cares just about maintaining output in its natural level, the variance of the real rate of interest is related only to the variance of the IS shock \( v \). This is because when \( y = y^* \) and \( v \) is zero, the real interest rate is always equal to the natural interest rate \( r^* \) (see equation of the generic IS curve (2)). Therefore, a central bank more concerned to maintain the inflation target generates a more variable real interest rate.

**CONCLUSIONS**

When the central bank minimizes a loss function depending on the variability of inflation and output, the determination of these variables (inflation and output) is completely independent of the parameters and the form of the IS curve. Fiscal variables, or other parameters showing preferences (some of them intertemporal) of the private sector, do not affect the levels and the variances of output and inflation at any moment.

In a kind of world as the one described, inflation and output will behave according to the preferences of the central bank. When this institution prefers more stable inflation than more stable output it will get what it wants: inflation will vary less but output will vary more. Ex post real interest rates will vary also more the higher is the preference of the central bank on the stability of inflation.

The way in which the central bank eliminates the influence of the parameters of the IS curve in the aggregate demand is through the optimizing Taylor rule, which is a rule in interest rates very similar to the original rule proposed by John B Taylor in 1993. In a IS function where fiscal variables matter (the old Keynesian IS curve), an expansionary fiscal
policy will increase the Wicksellian natural rate of interest. Then, according to the optimizing Taylor rule, the actual rate of interest must increase, which eliminates the influence of the fiscal policy on the aggregate demand. Starting in an equilibrium situation where output is equal to its natural value, and inflation is in its target, this automatic countercyclical policy will maintain output and inflation at their targets.

That central banks set rules to maintain targets in output and inflation is not a problem per se. The real problem is that the central bank has monopoly power in the creation of purchasing power and its preferences do not necessarily reflect social preferences. A central bank that praises too much the stability of inflation may induce a high variance in three very important real variables: output, employment and the real interest rate. Is that convenient for society? That is something that has to be discussed in the future.

REFERENCES


